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A Review of Optimal Investment Rules in Electricity Generation

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Abstract

This paper provides an introduction to optimal investment rules in electricity generation. It attempts to bring together methods commonly used in practice to assess electricity generation investments as well as the sophisticated tools developed by mathematical economists in the last thirty years. It begins with a description of the fundamentals of the problem (economic context of the energy and electricity sectors, the technical constraints and cost structures of generation technologies). In a second part it recalls the investment rule based on the positivity of the net present value (NPV) together with the standard tools of corporate finance needed to perform this evaluation (CAPM, WACC). This list is completed with the more specific tool of Levelised Cost of Electricity (LCOE) used by electrical utilities and policymakers. The third part of the paper shows how the advances made in the last quarter century by economic theory mainly under the real options trademark, challenged the standard investment rule. Using intensively stochastic control theory and its connection with partial differential equations, real options theory was able to assess the effects of key drivers of investment decision: uncertainty, time to build, competitive pressure and strategic interactions. The paper presents models that provided a breakthrough in the analysis of the impact of each of these drivers on investment decision rules. Despite this interest, the conclusion points out remaining obstacles for the adoption of these methods by financial divisions, mainly but not only their high level of technicity. Research guidelines that could help fill this gap are suggested.

*EDF R&D, 1 avenue du Général de Gaulle, 92 141 Clamart France and Finance for Energy Market Research Centre, www.fime-lab.org. rene.aid@edf.fr. This paper was written on the occasion of the Special Year on Financial Engineering for Energy and Commodity Risk Management and Hedging of Commodity Derivatives organized by the Wolgang Pauli Institute in Vienna in 2011. The author would like to give a special thanks to the organizers and in particular to Fred E. Benth and Peter Laurence. A special thanks also goes to Nicolas Langrené who helped me a lot improving this paper. Disclaimer: The views, assumptions and opinions expressed in this article are those of the author and do not necessarily reflect the official policy or position of EDF.

Contents

During their monopoly period, electric utilities developed computational economic models to assess their investments in generation. Those models relied on operations research methods (stochastic dynamic programming, linear programming, mixed-integer programming) together with an important effort in time-series analysis for long-term demand forecasting. The key words were "optimisation" and "planning". A perfect example of this approach can be found in the International Atomic Energy Agency expansion planning course (see International Atomic Energy Agency [1984]). In the early 19-80s with Chile setting the first example, the idea that markets can do a better job for electricity generation than monopolies began to spread around the world. Now 30 years later, competition for market shares, trading, risk management, electricity price modelling and even IPOs have become common in the power business. But the economic context is far from what was expected from liberalisation. Electricity prices are at their highest peak ever driven by expensive and volatile oil prices. Long-run resource adequacy is a concern for each European state and last but not least, global financial crises made the future more uncertain than ever. All those factors cast some doubt on the ability of former expansion planning methods to offer a suitable answer to cope with this new context of uncertainty and competition. Moreover, from a regulation point of view, many concerns were raised that the electricity market may not be able to provide the right signals at the right time to foster investment in electricity generation to ensure the desired level of reliability (see Joskow and Tirole [2007] for an introduction and some answers to the question). The increasingly complex situation of electricity markets led to a new interest in asset valuation methods and optimal investment rules.

Indeed, with electricity market liberalisation and trading activities development, it was no time before financial mathematics methods were applied to electricity generation asset valuation (see Pilipovic [1997]) and real options methods were promoted (see section 2.3 for a definition). But two observations led us to undertake this paper. First, at the recent exception known to the author of Fleten et al. [2007], all applications to real options for electricity generation assets valued the flexibility of the power plant and not the flexibility of the investment decision itself. The former corresponds to standard — but complex — net present value computation. The latter corresponds to the core of real options investment theory. Second, despite more than 30 years of development, real options investment still stands at the doorstep of financial divisions in general and in electrical firms in particular (Block [2007]). One reason that could explain why electricity generation assets are still evaluated using simple spreadsheets (see Vuorinen [2009] for examples) is due to the difficulties of embracing both the complexity of power market microstructure, plant characteristics, contraints and cost structure and the sophisticated valuation methods developed in the last decades based on financial mathematics.

This paper is an attempt to fill this gap and to provide a bridge between these two worlds to help both engineering economists and academic researchers to get the basics. It presents the main elements needed to enter in the field of optimal investment rules for electricity generation assets. A large body of textbooks on this subject already exists. But they focus on the power system aspects of the problem (see Khatib [2003], Vuorinen [2009] for example) and use standard economic and financial theory for evaluation purposes (net present value rule and weighted average cost of capital). The economic difficulties raised by competition, market imperfection, risk management issues and strategic interactions do not benefit from the important research performed in the last decades. This work gives special attention to the progress made by the theory of investment under uncertainty. It covers the results developed under the real options trademark and its related fields. They are based on the application of the various forms of stochastic control theory to toy models representing a large variety of market situations. This approach led to a deep understanding of optimal investment dynamics.

This work is divided in the following way. Section 1 covers the economic context of the energy sector (section 1.1), the technological aspects of electricity generation (section 1.2), electricity market fundamentals (section 1.3) and the investment problems faced by decision-makers (section 1.4). The decision-maker's investment toolbox is then addressed in section 2. It presents the Net Present Value rule (section 2.1) and the real options rule (section 2.3). It also explains what Levelised Cost of Electricity means and how it is used by electric utilities and regulators (section 2.2). Since all these tools require long-term prices to perform their valuation, section 2.4 presents the main market modelling methodology used to provide long-term insight into electricity prices. Section 3 presents the main economic models based on stochastic control methods that allowed the analysis of key drivers of investment in production assets. It covers the effects of uncertainty (section 3.1), time to build (section 3.2), competition (section 3.3) and strategic interactions (section 3.4). In most of those examples, the use of continuous-time stochastic economic models makes it possible for each of these drivers to deduce an explicit form for the optimal investment rule, allowing in-depth comparative statics. They show that the NPV rule as well as the real options rule are not systematically the right rules to apply. Section 4 uses this remark to draw conclusions on what investment rules should be applied to electricity generation and to propose some research prospects to help filling the gap between investment theory and practices.

1 The underpinnings of the problem

Investing in electricity generation has always been a challenge. It combines a substantial set of difficulties. The non-storability of electricity compels production to be adjusted on a real-time basis to consumption. This would be easy without the high level of uncertainties involved in both production (outages and inflows) and consumption (demand has a short-term weather dependency and a mid-term economic growth dependency). Moreover it is necessary to anticipate demand on a long-term basis to be able to satisfy the demand at all times, due to the long time it takes to build power plants. Electricity producers must choose amongst a wide range of very different technologies. They know that some plants are to be used nearly every hour of the year while others would be used only to produce during a small number of peak hours per year, making their return on investment very uncertain. Electricity market liberalisation has added competitive pressure to this already complex situation. Power producers are now competing for production and retail market shares. This section is devoted to these underlying aspects of the problem of investing in electricity generation. Since there is a growing dependency between the electricity sector, the energy sector and the global economy, section 1.1 presents the main drivers that are shaping electricity generation investments. Section 1.2 and 1.3 provide a description of the main available technologies (cost structure and operational constraints) and of the microstructure of electricity markets. Then section 1.4 gives a non-exhaustive list of decisions a utility must take when investing in electricity generation. Some historical and literature comments are given in section 1.5.

1.1 Economic environment

Five drivers are shaping energy's future: rising demand, the growing scarcity of fossil energy sources, the global warming risk, environmental and energy regulations and the financial crisis.

According to the International Energy Agency [2010a] New Policies Scenario[1], world electricity demand is expected to grow on average by 2% per year between 2008 and 2035, from 16,819 TWh to 30,300 TWh. This growth is mainly driven by non-OECD countries and in particular by China, India and Brazil. To meet global demand, production capacity should be increased by 5.9 TW; it is now 3.6 TW. In monetary terms, the needed investments correspond to 16.6 trillion USD2009. For the same period in time, if we focus on Europe[2], the numbers are still impressive. Electricity demand should increase by 0.6% on average per year from 3,339 TWh to 3,938 TWh. Because of old plant retirements, 800 GW of new installed capacity should be added. This corresponds to a financial investment of 1,712 billion USD2009. Those numbers correspond to a huge industrial and financial effort. But that is not all. According to World Energy Outlook 2010 scenarios for the same period of time, European Union electricity demand could reach 3,938 TWh (New Policies Scenario), but it could also be 3,771 TWh (450 Scenario) or even 4,094 TWh (Current Policies Scenario). In the 450 Scenario, 900 GW of new production capacity should be added. This huge difference with the level forecasted in the New Policies Scenario illustrates the importance of *demand uncertainty* faced by the electricity sector. And since power producers are no longer a monopoly now, they cannot hope to transfer any over-investment cost to the consumer. Mistakes can now lead to bankruptcies.

IEA scenarios represent the various efforts nations should make in order to reach the carbon emission level that would help avoid the possible dramatic outcomes of global warming. According to the Intergovernmental Panel on Climate Change [2007] report, if nations were to confine the Earth's temperature increase to below 2°C, greenhouse gas concentration should be kept under 450 parts per million of CO2 equivalent. Sir Nicholas Stern assessed global warming economic consequences in the Stern Review (Stern [2010]). The risks inherent to global warming are one important driver of the development of renewable and non-emissive energy sources.

The growing demand for fossil energies (oil, gas and coal) has already propelled oil and coal prices and volatilities to unseen levels. From 2003 to 2007, crude prices rose fourfold ($35 /bbl to $120 /bbl) and coal prices (CIF ARA API2) sevenfold ($30 /Mt to $200 /Mt). Figure 1 illustrates this extreme volatility in the period from March 2006 to March 2011 showing the new price rise after the 2008 crunch sent oil and coal prices and volatilities to unprecedented heights. This inflation finds its roots both in the belief that oil production will reach its global peak in the very near future[3] and in the current saturation of installed oil production capacities by emerging countries growing needs. An analysis by Büyükşahin et al. [2008, figure 10] shows that when crude oil prices were reaching levels of $100/bbl and above, non-OPEC less South Arabia spare production capacity was close to zero.

Governments are taking action to mitigate their dependencies on fossil energy and the effects of global warming. Their policies take the form of an increasing environmental and energy regulation pressure. But the general trend is to avoid outdated expansion planning methods and to prefer market mechanisms. In the case of Europe, during the period from 1992 (European electricity market creation) to 2010, every year has seen a new regulation policy that had a direct impact on electricity firms, making regulation an uncertainty factor for electric utilities.

[1] New Policies Scenario is defined as the expected energy demand and capacity growth when current environmental policies and announced regulations are taken into account. See International Energy Agency [2010a], page 46.

[2] European Union: Austria, Belgium, Bulgaria, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Romania, Slovak Republic, Slovenia, Spain, Sweden and United Kingdom.

[3] or even has already passed it. According to World Energy Outlook 2010 (p. 48), world oil production is to stay at 68 mb/d for the next 25 years.

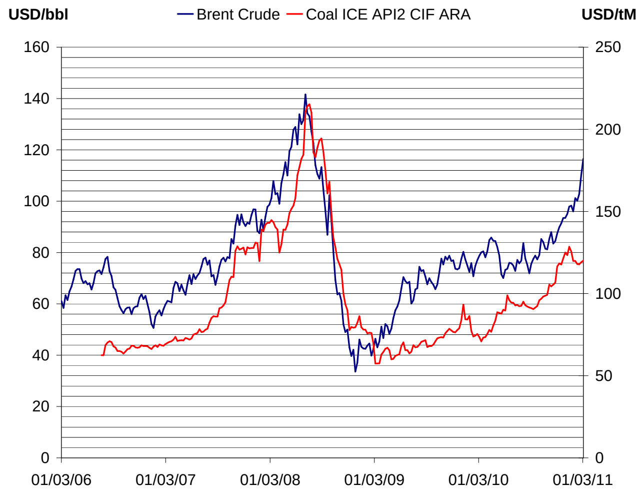

Figure 1: Oil WTI and Coal API 2 CIF ARA prices from March, 2006 to March, 2011.

Finally, the financial crisis started in mid-2008 and continues to this day with the sovereign debt crisis had a direct impact on the electricity industry through an important decrease in industrial consumption. According to the International Energy Agency [2009, chap. 3, p. 156] World Energy Outlook, electricity consumption of OECD countries fell by 2.6% in the last quarter of 2008 on a year-to-year basis. To give a more striking image of the impact of the crisis, one should have in mind that Germany's consumption fell by 6.5% in Q2-2009. Moreover, the possibility of an impending recession in Europe casts doubt on expected increases in electric consumption, in particular in the industrial sector.

1.2 Electricity generation technologies

A wide range of available technologies to produce power exists. They can be sorted into two broad families: thermal and non-thermal technologies. Thermal technologies burn a fuel to heat a fluid that spins a turbine, producing electricity. They cover:

- nuclear: Boiling Water Reactor (BWR), Advanced Boiling Water Reactor (ABWR), Pressurised Water Reactor (PWR), European Pressurised Reactor (EPR),
- gas: standard or combined cycles,
- coal: Conventional, Advanced, Gasification, with or without Carbon Capture Storage,
- diesel: oil,
- biomass-fired plants.

Non-thermal plants refer to technologies using a natural mechanical source of energy:

- wind: on and off-shore farms,
- solar: photovoltaic or concentrated solar power
- gravitational energy from flooding water, tides, etc

	Investment USD09/kW	O&M USD09/kW/y	TTB y	Lifetime %	Load Factor %	Efficiency
Gas	400 - 800	20 - 40	1 - 2	20 - 30	-	0.5
Coal	1,000 - 1,500	30 - 60	4 - 6	40	-	0.3
Nuclear	1,000 - 2,500	45 - 100	5 - 9	40 - 60	85	0.3
Wind onshore	1,000 - 2,000	15 - 30	1	20 - 40	15 - 35	0.3
Wind offshore	1,500 - 2,500	40 - 60	1 - 2	20 - 40	35 - 45	-
Solar PV	2,700 - 10,000	10 - 50	1 - 3	20 - 40	9 - 25	-

Table 1: Power generation technologies cost structure. source: International Energy Agency [2005, 2010b]. Investment: overnight cost; O&M: operation and maintenance; TTB: time to build.

When it comes to investment, a decision-maker faces a particular power plant choice with all its detailed specifications. But, for economic modelling purposes, it is not necessary. It is more useful to have in mind an order of magnitude for the cost structures of those different technologies and a broad idea of their technical characteristics. As hydrogeneration project cost highly depends on the topology of the region where it is to be built, we limit our scope to thermal power plants, wind farms and photovoltaic. Table 1 provides their cost structure. The investment cost is only the up-front cash-flow a utility has to pay to get the power plant built. It is sometimes referred to as *overnight costs*, since it is what one would have to pay if the power plant could be built in one night. Operation and maintenance costs (O&M) refer to employees' salaries and expenses required to maintain the plant in reliable production conditions. Lifetime is the given expected lifetime of the power plant provided by the builder. The load factor corresponds to the fact that a power plant is rarely expected to produce at its maximum installed capacity all the time. It is the ratio that should be applied to the installed capacity to get its expected production capacity[4]. For instance, even though onshore wind farms have a relatively low investment cost compared to standard coal-fired plants, their load factor is much lower. Efficiency corresponds to the power plant's thermodynamic efficiency. Given one unit of energy in heat form, efficiency tells us how much electric energy will be obtained. Gas-fired plants are the most efficient. Their efficiency is even expected to increase to 60% in the near future.

As it appears, investment costs exhibit a great variance both inside each family and from one family to another. At the extremes, the investment costs of a photovoltaic farm can be 10 times greater than that of a simple gas turbine. Finally, attention should be paid to the lifetime expectancies of electricity generation plants. Coal and nuclear plants and even wind farms are expected to last for 40 years up and running. For nuclear plants, a life expectancy of 60 years has already been admitted in the US. From an economic point of view, this long lifetime implies a non-negligible risk of experiencing a downturn period where fixed costs are no longer recovered. The British Energy bankruptcy in 2002 provides a concrete example of this situation.

Power production technologies highly differ in cost structure as well as in the service they can provide for load-following purposes and CO_2 emissions. Due to the increase in non-controllable

[4] Not to be mistaken for *capacity factor*.

	Start-up time min	Ramp Rate %/min	Min Stopping Time hour	CO2 emission t/MWh
Gas turbine	5 - 10	20	3 - 8	0.6
Gas Combined cycle	30 - 60	5 - 10	3 - 8	0.35
Coal	-		4 - 8	1
Diesel	1 - 5	40	2 - 6	
Nuclear		1 - 5	24	0

Table 2: Power generation technologies characteristics. source: International Energy Agency [2005, 2010b], Vuorinen [2009]

and highly variable wind and solar electricity production, *flexibility* is presently a major concern when assessing production technologies. Table 2 gives a few characteristics of thermal production plants. The start-up time represents the time needed to get the power plant from the cold to the hot state. Then, to reach its full available capacity, one has to deal with the ramp-up rate. For instance, gas turbines can increase their output at a rate of 20% of their maximum capacity per minute. When a power plant is shut down, it cannot start up again immediately. It has to be cooled down and maintained a certain time still before it can be used again. Lastly, emission rates highly differ between the most emissive technology (coal-fired plants without sequestration, 1 t/MWh) and no-emission thermal technology (nuclear). Regarding flexibility, it should be noticed that combined cycle gas turbines can rapidly increase their production but the warm-up time is much longer than for standard gas turbines.

1.3 Electricity markets

Investors in electricity production can hope to get cash-flows from four markets: the spot market, the forward market, the balancing market and the retail market. The spot market corresponds in general to the day-ahead hourly market. It is commonly the main market on which power plants are being evaluated. In almost all countries where it has reached its full development, it presents the same characteristics: strong patterns of daily, weekly seasonal and yearly use driven by weather conditions and the overall economic activity, spikes, and sometimes negative prices. For a deeper description of electricity markets, the reader can refer to classic textbooks such as Géman [2007], Clewlow and Strickland [2000] or to Géman and Roncoroni [2006].

The forward market is to be considered a hedging tool. But due to the non-storability of electricity, it presents a particular structure exhibiting *sparseness* and *delivery periods*. To avoid dilution of liquidity, only a few forward (or futures) contracts are generally available on the market. For instance at EEX, the German power exchange, the calendars for the next six years, the next eleven quarters, the next nine months and the next four weeks, at one day in time are opened. Each contract is quoted in two forms: base-load (delivery each hour of the given delivery period of the contract) or peak-load (delivery from Monday to Friday from 8am to 8pm). But only the next three calendars, the next eight quarters and the next five months are being exchanged. For a description of the electricity futures market, the reader is referred to Benth et al. [2008].

The balancing market is specific to electricity (and gas) markets due to the need for the system operator to adjust production to consumption on a real-time basis. This market micro-structure highly depends on the country and on the regulation. But its general idea is to allow the system operator to buy the lacking production from load-serving entities (possibly producers but also consumers) or to sell the excess ouput. Balancing markets produce two kinds of prices: prices to increase production and prices to decrease production. They serve a second purpose: they provide indexes used to assess imbalanced energy bills for each load-serving entity.

The retail market encompasses both the industrial and domestic consumer markets. Prices follow a much smoother pattern even when they are not subject to government regulation. For an overview of the relationship between retail and wholesale market prices, the reader can refer to the description of the Scandinavian market provided in von der Fehr and Hansen [2010].

1.4 Decision-maker's problem

Power companies have different investment opportunities depending on their size. There is little comparison between the possibilities offered to a small entrant in the retail market and a historical player owning a large portion of the installed production capacities. Due to more difficult access to capital markets, a small entrant is confined to a set of production investment possibilities involving wind farms, gas turbines and small hydrogeneration. In the opposite case, the set of opportunities for a large player includes all available technologies. Nevertheless, decision-makers in both kind of companies try to answer the following questions:

— should I invest in electricity production or should I supply my client with the wholesale markets?
— in what kind of production asset should I invest?
— how much of each kind should I own?
— should I do it now or should I wait?

These questions boil down to a single one "*How should I choose between the different available investment projects?*"

In a large electric utility, each year brings an important set of new investment opportunities. One may naturaly think of building of new production capacities. But this also concerns existing power plants for which important investment decisions can be taken. Replacing parts of the turbine or of any main parts that could have a direct impact on production efficiency can still involve substantial upfront costs. In the event of a recession or economic downturn, decision-makers may have to consider closing a power plant before the end of its lifetime. Intermediate decisions can involve mothballing the plant, i.e. closing but not dismantling it, so it can be used again in several years when times are better.

And beyond choosing amongst a set of mutually exclusive possibilities, decision-makers also face the problem of deciding which *portfolio* would be the best. Would a combination of a little hydrogeneration, coal plants, wind generation and solar photovoltaic be a better option than investing everything in just one technology? Is it possible to consider that there is a portfolio effect in mixing the different available technologies? Or should they consider buying or selling the production shortfall or surplus from or to the market?

The answer to these questions heavily relies on the objective the company is assigned. Since most electric utilities are now quoted, their goal is no longer to maximize social surplus as they used to do during their monopoly period, but to *maximize the value of the firm*. Investment valuation in electricity generation should be done with this background in mind.

1.5 Commented references

A global assessment of 30 years of worldwide liberalisation of the electricity industry can be found in Sioshansi and Pfaffenberger [2006]. This assessment does not provide economic evaluation of benefits and costs of deregulation but gives qualitative issues involved in the different countries where deregulation has been implemented. Bunn [1994] conducted an evaluation of electricity market reforms in England and Wales. A more recent one for the electricity market reform process can be found in Sioshansi [2006]. A reader interested in the evolution of the investment problem for electric utilities can look at the author's paper Aïd [2010]. Those interested in global warming and its economic implications can spend a little time reading the Stern's Review (Stern [2010]) or Houghton [2009]'s beautiful monograph with a large set of photographs illustrating the dramatic effects of natural disasters. Information power plants' costs and technical performances is hard to come by. But one can rely on reports provided by the International Energy Agency International Energy Agency [1998, 2005, 2010a, 2008, 2010b] and the Nuclear Energy Agency (Nuclear Energy Agency [2000]) for a broad overview of the relative costs of production technologies. Kaplan [2008] special study made upon the US Congress's request also provides detailed information on power plant cost structures. The Danish Energy Agency has compiled and made public a comprehensive, up-to-date, detailed report on financial and technical power plants data (Danish Energy Agency [2010]). Finding detailed power plant characteristics for a whole country is a rare event enough to mention that the Commission for Energy Regulation together with the Northern Ireland Authority for Utility Regulation has published a detailed database of all the power plant characteristics in Ireland for a project named *All Island*. Data can be found at the url `www.allislandproject.org`. Books on electricity markets are legion now. But few deal with electricity prices modelling. For an introduction to energy finance, the reader can consult the main monographs which are Clewlow and Strickland [2000], Eydeland and Wolyniec [2002], Pilipovic [2007], Benth et al. [2008]. Lastly, since this is presently a major concern of power system regulation, a reader looking for a quantitative definition of flexibility as well as an introduction to this problem can read Lannoye et al. [2010] and the reports provided by the North American Electric Reliability Corporation [2010].

2 The decision-maker's toolbox

The purpose of the preceding section was to emphasize the complexity of the problem of making investment decisions in electricity generation, so that the baseline methodology companies use to evaluate their choices would appear in contrast a simple rule of thumb. Indeed the core of the decision-maker's method is to compare the present costs with the expected future net benefits of any investment project. If the expected future benefits exceed the present costs, then do it! This rule is known as the net present value rule (NPV) and is presented in section 2.1 in detail to show that it is not as simple as it looks. We stress in particular the difficulties linked with the fact that future benefits have to be *discounted* to take into account the time-value of money and that discounting is also used in practice to take risk into account. Section 2.2 focuses on an economic indicator specific to the electricity sector, the Levelised Cost of Electricity, which is derived from the net present value. It gives the constant price above which the net present value of a given investment project is positive. This indicator is a great concern in regulation and energy policy discussions as well as long-term contract negotiations. There had been no alternatives to the NPV rule until the mid-1980s, when the real options rule was proposed. It is described and commented on section 2.3. In both cases, long-term expected prices for electricity are needed to compute the NPV of a given project or to apply real options methodology. Section 2.4 presents the most common long-term price models used by electric utilities. They are mainly *static* long-term equilibrium models. We leave aside alternative approaches relying only on simulation principles such as agent-based models or *dynamical system* models. Even though this approach has been developed to offer an alternative to the growing complexity of the electric system, they are beyond the scope of this review. We refer to Ford [1997] and Foley et al. [2010] for an introduction and a review of these alternatives. Lastly, section 2.5 gathers remarks on the history of and the literature on the subject.

2.1 Net Present Value

Every business school or corporate finance master's degree student is now being taught that an investment should be undertaken if its costs do not exceed the *expected discounted* total revenue. Consider a project with an overnight cost of I that is expected to yield the net revenue (income minus operating cost) f_t for the year t during its lifetime T. The investment cost is supposed to be certain and the net revenues are supposed to be uncertain. The project is assumed to have no terminal value and no decommissioning costs. The Net Present Value (NPV) rule states that the project should be undertaken if and only if the present value of the project is greater that the investment cost:

$$\mathbf{V} = \mathbb{E}\left[\sum_{t=1}^{T} \frac{f_t}{(1+\rho)^t}\right] - I \geq 0. \tag{1}$$

Albeit apparently simple, this rule involves three major difficulties: the determination of net revenue f_t, the determination of the probability measure to compute the expectation and the determination of the discount rate ρ. Net revenues depend on the project and on the decision-maker's knowledge. There is no general theory about them. This is not the case for the probability measure. If the net revenues f_t were to depend only on traded assets, the asset value theorem would apply and one could use both the risk-neutral probability and the risk-free rate (Duffie [2001]). But corporate investment deals with non-financial investments for which in general some risks have no market price. This is the case for the main risks involved with electricity generation: inflows for hydrogeneration, wind for wind farms and unplanned outages for any generation plant. Hence, in this case, the market is incomplete and it is not possible to define a unique risk-neutral probability to perform an evaluation that would be independent of market participants' preferences towards risk. There are different ways to take those preferences into account. The onecorporate finance divisions use is not necessarily the one that would have received economists' preference. In practice, *risk is taken into account through the discount rate*, which becomes a risk-adjusted discount rate. The most commonly used corporate finance textbooks, such as Grinblatt and Titman [2002, chap. 11] and Brealy and Myers [2007, chap. 9], supports this method, which is based on a standard arbitrage argument. If it is possible to find a financial portfolio perfectly correlated with the project's cash-flows, then one should use the expected return of this replication portfolio to discount the project's cash-flows. The criticism of this method is that it uses the same parameter to express two different things: time preferences and risk preferences. The theoretically correct way of dealing with the risk of a project would be to use utility functions and to compute the *certainty equivalent* of the risky cash-flows. Even though these methods are deemed equivalent in standard corporate finance textbooks (see again Grinblatt and Titman [2002, chap. 11] and Brealy and Myers [2007, chap. 9]), the certainty equivalent method is never cited as a method of choice by Chief Financial Officers, who always prefer to use the discount factor to take risks into account (see Graham and Harvey [2001]'s survey).

Thus decision-makers try to find the risk-adjusted discount rate that reflects the risk of the project. But it is very unlikely to find a portfolio of financial assets that would perfectly replicate the cash-flows of the project. In fact, if such a portfolio were to exist, it would mean that the market considered is complete. Thus, decision-makers are pushed to a less ambitious method. They are reduced to using the discount rate that will allow the firm to pay back its financial resources, both debt and equity. This leads to the identification of the right discount rate with the *weighted average cost of capital* (WACC):

$$\rho = \frac{D}{D+E} r_d + \frac{E}{D+E} r_e, \tag{2}$$

where D is the firm's level of debt, E the level of equity, r_d the expected return of the debt and r_e the expected return of the equity. In this relationship, the only variable that presents a difficulty is the expected return of the equity. It is not directly observable by the decision-maker. Its estimation relies on the idea that financial markets should be in equilibrium: investors should expect a return that would compensate the risk of the project. The main financial market equilibrium model used is the Capital Asset Pricing Model (CAPM) first developed by Sharpe [1964]. It states that the expected return r_i for the financial asset i satisfies at the equilibrium:

$$r_i = r_f + \beta_i (r_m - r_f), \tag{3}$$

where r_f is the expected return from a risk-free financial asset, r_m is the expected return from the market portfolio and

$$\beta_i = \frac{cov(\mathbf{r}_i, \mathbf{r}_m)}{\sigma_m^2} \tag{4}$$

σ_m^2 is the variance of the return of the market portfolio. Bold letters are used here to designate random variables. The CAPM states that an investor is expecting an excess return over the risk-free rate that is proportional to the market risk premium. The more the financial asset return $\mathbf{r}_i$ is correlated with the market returns $\mathbf{r}_m$, the higher the expected return. The investor is only expecting to be rewarded for the *systematic* risk of the project, i.e. the risk that cannot be cancelled out by a well-diversified portfolio. The simplicity of the CAPM makes it a preferred tool by corporate finance divisions despite its known limited performance in predicting expected returns. The fact that the CAPM fails to explain the expected returns of common stocks was statistically established in the 1990s by Fama & French (Fama and French [1992, 2004]).

Due to its overall importance in project selection (it acts like a threshold that a project has to cross to be undertaken), the WACC focuses decision-markers' attention and is subject to two main controversies.

The first concerns the way risk is taken into account in the evaluation process. We have already said a few words above on that point. We will not develop this point again here, but it will naturally come back in the section onthe optimal investment rule (section 3). The other point that raises many issues is the problem of the granularity of the discount rate inside the firm. Despite the prescription of the theory that the discount rate should reflect the risks of the project, most companies tend to use a firm-level discount rate. Sometimes, if the firm is divided into very separate divisions on well-separated markets, it will probably use a discount factor for each business unit. But there is evidence that even in that case, firms are still reluctant to apply differentiated discount factors. Graham and Harvey [2001]'s survey shows that Chief Financial Officers widely use discount rates determined at the corporate level despite known evidence of bias resulting from this practice, as recently evaluated (Krueger et al. [2011]). The rationality behind this apparently suboptimal behaviour can be explained by what economists call *influence costs*. As discussed in Martin and Titman [2008], these costs reflect the time and effort managers supporting the project devote to justifying a lower discount rate and the time and effort managers performing the evaluation devote to estimating this bias. Managers supporting a project get personal benefits from seeing it realized. Benefits can be linked to project realization or the manager can experience personal satisfaction in developing large projects. Introducing different discount rates for different business units may open the door for a premium for more persuasive managers, spending time and effort in increasing their political influence in the firm to obtain a lower discount rate. Using a single discount factor may on the contrary show that managers should not focus their effort ontheir projects's financial aspects.

Lastly, one could ask whether an investment procedure based on the NPV rule together with an estimation of the risk-adjusted discount rate based on the CAPM leads to its desired outcome, bringing back an expected return equal to the discount factor. Economic and finance literature exists that studies the reliability of an investment process based on NPV and expected rate of return. For public investments, the World Bank has published different studies showing that there is a negative bias between *ex-ante* expected rate-of-return and *ex-post* realized rate of return Pohl and Mihaljek [1992]. The authors' analysis showed that the mean average expected rate-of-return of a sample of more than 1,000 projects between 1974 and 1987 was 22% when the realized rate-of-return after completion was only 16%. This negative bias is also confirmed in private sector investments (see Statman and Tyebjee [1985] and the references therein for specific studies of various private sectors). As an example concerning electricity generation, Allen and Norris [1970] report an underestimation of 45% of R&D projects but with a strong positive skewness that can lead to an 800% overrun. Quirk and Terasawa [1986] report nuclear power plant construction cost overruns in the late 1960s that could reach four times the estimated costs. Nevertheless, one should be aware that even if the forecasts for expected cost and profit of a sample of projects are not biased, realized projects can exhibit a negative bias. The reason for this is that only projects with positive net present value were selected and those are mainly projects for which costs were underestimated and profits overestimated. This result has been known since Brown [1978] and Miller [1978].

We conclude this section by showing how the NPV rule translates into the case of the evaluation of an electricity generation project. Armed with these three tools (NPV, WACC and CAPM), our decision-maker is ready to analyse the economic benefits from different electricity generation projects. Let us see the computational difficulties involved in performing the economic evaluation of a generation asset. For a power plant whose production is entirely sold on the spot market, its net present value takes the form:

$$\mathbf{V}_g = \mathbb{E}\left[-\sum_{t=0}^{T_c-1} \frac{I_t}{(1+\rho)^t}\right] + \sup_q \mathbb{E}\left[\sum_{t=T_c}^{T_c+T_l-1} \frac{g(q_t)-\kappa_t}{(1+\rho)^t}\right] \tag{5}$$

where T_c is the random construction time, T_l the power plant lifetime, κ_t the Operation & Maintenance cost, g the short-term cost function of the power plant and q_t its production level at time t. The production level q_t belongs to a non-convex time varying random set of constraints. Non-convexities arise from dynamic constraints and minimal production levels (see section 1.2). Randomness comes from unplanned outages and technical problems that may reduce the available power. One should note that the present value of the power plant in relation 5 is obtained by solving a stochastic control problem. If dynamic constraints, minimal production levels and start-up costs are neglected, the stochastic control problem takes a much simpler form since the optimal control is a bang-bang solution, i.e. the power plant produces at its maximum level each time its marginal production cost is lower than the electricity spot price. Nevertheless, even with this outrageous simplification of the problem, there are still many difficulties left. In this case, the present value of a power plant of one megawatt takes the form:

$$\mathbf{P}_g = \mathbb{E}\left[\sum_{t=1}^{T_l} \frac{u_t(S_t^e - h_t^f S_t^f - h_t^c S_t^c)^+ - \kappa_t}{(1+\rho)^t}\right] \tag{6}$$

where $u_t \in \{0, 1\}$ is a random process indicating if the plant is available and S_t^e is the electricity spot price, S_t^f is the fuel cost and S_t^c is the CO_2 emission price. The coefficients h^f and h^c are respectively the heat rate and the emission factor. When dynamic constraints are neglected, a power plant appears as a strip of call options on the clean fuel spread (gas, coal or oil). Its evaluation requires the three-dimensional joint modelling of electricity, fuel and CO_2 prices. Alos et al. [2011] recently provided an asymptotic analytical formula for this three-asset derivative when all prices are assumed to follow correlated geometric Brownian motions. Regarding the joint price model to use, one difficulty stems from the long-time horizon involved in the net present value. It considerably exceeds market horizon. Market horizon is three years for electricity futures, three years for CO_2 emission prices and five years for fuel prices, whereas we have seen that the expected lifetime of a coal-fired plant, for instance, is 40 years. Section 2.4 will present the main method electric utilities use to obtain long-term electricity prices. Thus even with crude simplifications of its operating constraints, the economic evaluation of a power plant still presents major difficulties. But neglecting dynamic constraints can be a concern. All other things being equal, dynamic constraints can have a large impact on a power plant's valuation. For an example, using mathematical methods that are beyond this review, Porchet et al. [2007] show that in an economic situation where flexibility matters (i.e. electricity prices are close to the proportional cost), the value of the plant over one year can be as much as 25% lower when considering dynamic constraints.

2.2 Levelised Cost Of Electricity

It is certainly to avoid the complexity of mathematical methods involved in power plant valuation that decision- and policy-makers rely on a much simpler economic indicator to assess the relative costs of different electricity generation technologies. The Levelised Cost Of Electricity (LCOE) is the minimum constant price of electricity leading to a null net present value. With one more step of simplification compared to (5) and (6), the net present value can be written as a function of the constant price of production p as:

$$\mathbf{V}(p) = -I + \sum_{k=1}^{T} \frac{N \cdot (p - h \cdot S - e \cdot S_c) - \kappa}{(1+\rho)^k},$$

where the investment cost I is supposed to occur only in one period of time, N is the number of hours per year the plant is expected to be running, h is its heat rate, S is the supposedly constant fuel cost, e the emission rate, S_c the price of CO_2, and κ the fixed cost. The LCOE p^* is then given by $\mathbf{V}(p^*) = 0$. With the expression above of the NPV, one has:

$$p = h \cdot S + e \cdot S_c + \frac{\kappa}{N} + \frac{1-\beta}{\beta \cdot (1-\beta^T)} \frac{I}{N},$$

with $\beta = 1/(1+\rho)$.

Example: First consider a coal-fired plant with an investment cost of 1,500 kUSD/MW, O&M 60 kUSD/MW/year, a lifetime of 40 years, running 3,000 hours per year (semi-base load) with a 40% efficiency rate and a 1 MT/MWh emission rate, coal price 90 USD/MT[5], CO_2 price €15 /MT, nominal discount rate of 10% and euro dollar parity. One gets a LCOE of 113 USD/MWh. For an onshore wind farm with an investment cost of 1,500 kUSD/MW, O&M 20 kUSD/MW, 40 years of lifetime, a load factor of 20% and the same discount rate of 10%, one gets a levelised cost of 100 USD/MWh. □

Considering our toy examples of a coal plant and a wind farm, and assuming that spot prices are observed to be on average above 100 USD/MWh, should coal-fired plants be replaced by wind farms? LCOEs make the hypothesis that a generation asset is always able to produce at that price level. If wind farm production were to be financed by selling its output to the spot market only, it would be necessary to take into account the fact that its production is not controllable and it may not be able to produce when prices are high. LCOE highly depends on the hypothesis made even on the restricted number of variables used above. The International Energy Agency [2010a] (figure ES.2, p 19) reports for instance that for Europe, coal-fired plant LCOEs can vary from 80 to 140 USD/MWh at a 10% discount rate. This variability would be harmless if technologies were clearly ranked. But for the three main technologies that can currently provide baseload or semibaseload power (nuclear, coal and gas), the ranges of LCOE do intersect substantially, ranging from 80 to 135 for nuclear, 80 to 140 for coal and 85 to 120 for gas.

Moreover, despite the sensitivity of the LCOE to the different variables and in particular to the discount rate, it provides important information for environmental regulation. Indeed, LCOEs give a direct estimation of the level of subsidies needed for technologies not yet profitable. According to the same IEA study, at a 10% discount rate, onshore wind farms are expected to be profitable between 120 and 230 USD/MWh. This provides an idea of the level of subsidies required for wind farms if the market price is under its LCOE. Lastly , LCOE of a given power plant building project is the basis for negotiation of long-term contracts. If production is sold at that constant price level, the investors can have a certain confidence that their costs will be recovered. More details on that subject can be found in Joskow [1985]. For a recent update of nuclear plant levelised cost performed by an academic institution, one can refer to Du and Parsons [2009] which also provides the spreadsheet used to perform the estimates.

[5] 1 MT of coal contains broadly 8.2 MWh of heat.

2.3 Real options

The NPV rule states that an investment is to be made as soon as its present value exceeds its costs. The real options rule challenges this point. It states that *if the decision-maker can wait and if the investment is irreversible*, then the investment should not be undertaken according to the NPV rule. It should be evaluated according to a rule that values this option to wait. If there is an opportunity to wait, then the decision-maker should use this freedom as a control variable to increase the firm's value to its maximum. A classic example to illustrate the difference between the NPV rule and the real options rule is a two time-steps model that can be found in Pindyck [1991] or Dixit and Pindyck [1994, chap. 2]. Consider investing in a widget factory that will produce one widget per year forever at no cost. The investment is irreversible and the decision-maker has time to take her decision. No one is expected to preempt this opportunity from the decision-maker. The investment cost is \$800. Prices for widgets are expected to be \$150 with probability 1/2 and \$50 with probability 1/2 and then to stay constant for ever. The discount rate is supposed to be 10%. Should an investment in that factory be made?

Following the NPV rule, one has:

$$-800 + \sum_{k=0}^{\infty} \frac{1/2 \times 150 + 1/2 \times 50}{1.1^k} = \$300 > 0$$

and the investment should be made. But, is it the maximum value that the decision-maker can extract from this project? No, because she has not taken into account the fact that her investment decision can be postponed. In fact she is in a situation where she should compare mutually exclusive investment alternatives: investing today or in one year. If she waits one year, she will invest only if the price of widgets is going to be 150 (in the other case, the NPV is negative). The factory's present value is then:

$$\frac{1}{2}\left[-800/1.1 + \sum_{k=1}^{\infty} \frac{150}{1.1^k}\right] = \$386 > 0.$$

The project value is higher in the case when the opportunity to wait is explicitly used as a decision variable. Thus the conclusion here is that the decision-maker should wait one year.

In this intensively cited example, the fact that the investment criterion has been changed does not appear completely clearly. It is more striking in McDonald and Siegel [1986]'s seminal paper that started the real options literature and is developed in detail in section 3.1. But let us already examine this point now. Consider an irreversible investment with a cost of I that brings access to a *present value* V. The value V is the expected discounted sum of all the future cash-flows produced by the investment. It is assumed that both investment I and value V vary over time so that $I = I_t$ and $V = V_t$. The NPV rule states: invest as soon as $V_t - I_t \geq 0$. The real options rule states: invest at the first time τ such that

$$\sup_{\tau \geq 0} \mathbb{E}\left[e^{-\mu\tau}(V_\tau - I_\tau)\right]. \tag{7}$$

The intuition behind this criterion is less obvious than for the NPV rule. What does it say? First, it states that the firm's objective is not to create but to *maximise* value. Thus if there is a possibility to postpone a project, this decision variable should be used to extract all the value.

Moreover, it says that there is a trade-off to be found between waiting for the difference $V_\tau - I_\tau$ to become as great as possible and seeing this difference being crushed down by the discount factor. Lastly, it is not easy to understand how to apply the relationship (7). The stopping time τ is a random process and one may have the feeling that the law or the strategy defining this stopping time is complex. The surprising and beautiful result is that the problem leads to a simple mechanical rule. For instance, in the case when V_t follows a geometric Brownian motion with parameters α and σ and $I_t \equiv I$ is assumed constant, McDonald and Siegel [1986] analytically describe the solution of the optimization problem (7). One should invest whenever V_t is above the value V^* defined by

$$V^* = \frac{n}{n-1} I, \tag{8}$$

where

$$n = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2\mu}{\sigma^2}} \tag{9}$$

In this case, applying the real options rule is simple: Compute the threshold V^*, compute the present value V_t every day and once it reaches or exceeds the threshold V^*, undertake the project. Taking for instance standard parameters for the present value process, $\mu = 10\%$, $\alpha = 4\%$, $\sigma = 15\%$, one has $n/(n-1) = 1.96$. The real options rule provides a threshold that is nearly twice the investment cost I. Hence, moving from the NPV rule to the real options one may substantially change the investment behaviour of power producers.

Real options methodology has met a huge success in the academic community since McDonald and Siegel [1986]'s seminal paper. Some statistics may help appraise the extent of this success. The exact query "real options" in title returns more than 1,300 papers in the EconLite full text economic database. In Marco Dias's website devoted to Real Options[6], a bibliography of papers and monographs on real options contains 2,600 references. Real options methodology has been applied to every possible industrial or managerial context, from natural resources exploitation to R&D project evaluation.

But this success has not been followed by its counterpart in the industry. The financial literature regularly provides inquiries on CFOs investment decision processes. Some were done before the rise of the real options principle (Gitman and Forrester Jr. [1977], Gitman and Mercurio [1982], Moore and Reichert [1983], Stanley and Block [1984]), some after (Fisher et al. [1989], Pinegar and Wilbricht [1989], Bierman [1993], Sangster [1993], Epps and Mitchem [1994], Trahan and Gitman [1988], Shao and Shao [1996], Bodnar et al. [1998], Graham and Harvey [2001], Baker et al. [2011]). Recent surveys still report CFOs overwhelming preferences for the NPV rule (more than 75% in Graham and Harvey [2001]) against the real options method, which received a 25% score in the same survey where CFOs were asked what capital budgeting techniques they were using. The real options rule appears in both Graham and Harvey [2001] and Baker et al. [2011] at the bottom of capital budgeting techniques' rankings whereas NPV stands at the top.

This impressive gap between the academic taste for real options and its low level of implementation in the industry raises the question why. There are already so many papers applying real options methods to different industrial sectors, like Moore and Reichert [1983], Busby and Pitts [1997], Damodaran [2000], Triantis and Borison [2001], Ryan and Ryan [2002], Brounen et al. [2004],

[6] http://www.puc-rio.br/marco.ind

Borison [2005], Block [2007], that the argument that it cannot apply to a given investment problem does not stand. In Baker et al. [2011] CFOs were asked why they used one technique more than another and why they did not use real options. The reason that emerges was the complexity of real options methodology. Indeed, we will see in section 3 that the required mathematical background to perform a real option analysis of an investment valuation is of an order of magnitude compared to NPV or LCOE. Moreover, depending on the situation and the model, one can obtain different decision rules. The clear-cut real options decision rule stating that for an irreversible investment one should take into account the option value to wait becomes hollow when the modelling of the uncertainties is changed, or when time-to-build, competition or strategic behaviour is introduced.

Also, it may be argued that capital budgeting methods implemented in practice by CFOs may be suboptimal, but simple rules that translate the difficult optimal control problem induced by the real options criterion. Indeed, asGraham and Harvey [2001] stated, the NPV rule is often completed by constraints on the internal rate of return, the ratio between the NPV and the investment (profitability index) or the payback time. These added constraints are shown to mimic the real options rules in simple cases (Dixit [1992], McDonald [1998], Boyle and Guthrie [2006]).

To conclude this section on real options, let us stress that the right question one should ask is if the currently few firms that use real options are more successful than firms using standard NPV. This question raises some methodological issues on defining what is a firm using real options and measuring the relationship between its implementation of real options and its performance level. To the best of the author's knowledge, Driouchi and Bennett [2011] is the only paper that tries to answer this question. Its analysis is based on the idea that a firm's use of the real options method by can be estimated by using public data to construct an index showing firm's awareness. The authors test the hypothesis that real options awareness procures a competitive benefit on a sample of 101 multinational corporations. Indeed multi-nationality provides a natural hedge for corporations against country's potential economic slowdown. In this context, a real options awareness could provide an excess of efficiency in using multinational investment opportunities. The authors find an excess negative relationship between multinational corporations' level of awareness of real options methods and their downside risk: the more multinational firms are aware of the real options value of investments, the less they are prone to downside risk. As the authors themselves point out, repeated studies have yet to confirm this finding since it is based on a small unique sample.

2.4 Long-term electricity price models

In section 2.1, we have seen that whichever valuation method is applied (NPV or real options), a long-term price model is needed to provide electricity and fuel prices. Although very popular in economics and finance literature on asset valuation, exogenous price models based on time-series analysis or stochastic processes are seldom used by utilities. Given the time-scale entailed by the lifetime of power plants, electric utilities mainly rely on electricity market equilibrium models. The equilibrium is defined by the least total cost needed to meet demand over a certain time horizon and with a certain reliability requirement. The total cost is composed of the exploitation cost and the investment cost. Reliability requirement refers to a loss of load probability being lower than a certain threshold. Since probability constraints are difficult to treat directly in an optimisation problem, they are often treated in a second step and a variable is introduced to take into account and to value the non-served energy. An example of the optimisation problem to be solved can be given in the following form:

$$J(D) = \min_{Q,L,I} \mathbb{E}\left[\sum_{t=1}^{T} \beta^t \left[\sum_i c_i \xi_{i,t} + g(X_t, D_t)\right]\right]$$

$$L_t + \sum_i q_{i,t} = D_t, \quad \sum_i x_{i,t} - q_{i,t} \geq R_t, \quad x_{i,t+h_i} = x_{i,t} + \xi_{i,t}, \quad Q_t \in \mathbf{A}_t(X),$$

where $X_t = (x_{1,t}, ..., x_{n,t})$ is the vector of installed capacity, $Q_t = (q_{1,t}, ..., q_{n,t})$ the vector of production level satisfying $q_{i,t} \leq x_{i,t}$ and belonging to a non-convex set $\mathbf{A}_t(X)$, D_t the demand forecast, R_t the reserve requirement, L_t the Energy Not-Served, $I_t = (\xi_{1,t}, ..., \xi_{n,t})$ the investments in production capacity, c_i the investment cost for production technology i, h_i the construction time for the technology i and β the discount factor. In this formulation, the function g represents the operating cost at time t for the installed capacity vector X_t. The operating cost's dependency on fuel prices, emission prices and cost of non-served energy is hidden in the short-term cost function g. The decision-maker gets the investment policy I_t as an output. This policy is used to assess the total capacity requirement of a given electric system. The long-term electricity prices needed to perform an economic valuation of generation projects are given by the *short-term marginal cost* of the optimisation problem, the derivative of g with respect to D.

Electric utilities still intensively used this approach to assess long-term electricity prices. In the past 30 years it has been used in the context of expansion planning studies. For interested reader, International Atomic Energy Agency [1984] provides a guide covering all the aspects of this methodology. It has led to the development of a series of software programs based on numerical optimisation for generation expansion planning. A detailed and exhaustive survey of those techniques, models and softwares can be found in Foley et al. [2010].

With the liberalisation process of the electricity system occurring in almost all countries, questions were raised about the soundness of continuing to use an approach that is so disconnected now from market reality. As we have seen in section 1, the first disconnect is the high level of uncertainty on fuel and emission prices. Power plant valuations greatly depend on the spread between spot price and fuel cost plus emission cost. With a high level of volatility together with great uncertainty about the long-term level of fossil energy prices, firms are generally doomed to perform *prospective* analysis, which consists of extracting a set of a few scenarios that are seen as a possible global economic equilibrium. They are defined by a small number of economic parameters, including economic growth, inflation rate, average crude oil price, average coal price, average emission price and energy and environmental regulation trends... Once these scenarios have been defined, an optimization problem as above can be formulated and solved to provide long-term electricity price trajectories for each one.

Even though they present challenging modelling problems, the increasing volatility of fuel prices and the introduction of emission permits are not the main drivers casting doubt on the pertinence of the approach above. A first difficulty that was already present during electric systems' monopolistic period is that the preceding approach, when formulated as a stochastic optimisation problem only provides a *policy*, i.e. an investment program that is not adapted to the realization of demand. In section 3.2, we will see an example of a formulation of a long-term investment model that provides the full investment strategy adapted to the realization of demand. One faces a second set of difficulties when trying to introduce competition and financial risk in the expansion planning method. The introduction of competition in electricity generation capacity expansion has led some authors to

Nash equilibrium models (Murphy and Smeers [2005]). The tractability of such models is an issue when a realistic situation is under consideration. In section 3.3, we will see how dynamic Nash equilibrium models can be formulated and solved. Risk was first introduced using utility functions or risk measures such as mean-variance criteria (Roques et al. [2008]). It was only recently that attempts were made to introduce discount factors differentiated by technologies (Ehrenmann and Smeers [2010]).

2.5 Historical and literature comments

The modern theory of investment decision goes back to Irving Fisher's treatise on interest rates Fisher [1930]. In this work, Fisher clearly stated the net present value rule but also the real options rule. Since his book is 600 pages long, we can specify that this is done in Chapter VII entitled *The Investment Opportunity Principles*. Firms ignored this work until the development of business schools and the establishment of a standard corpus of corporate finance textbooks such as Grinblatt and Titman [2002] and Brealy and Myers [2007]. But a significant improvement in its mathematical formulation had been made in between. When reading Fisher's treatise, one is confounded by the fact that the ideas are there but lack the suitable mathematical tools to provide their full insight. It was ignored by firms but not by economists. Real options methodology was clearly described with a more modern mathematical formulation by Marglin [1967, 1970]. In this paper, the fact that the option should be privately owned to keep its value was explicitly stated.

The idea that real options in the economy exist is clearly developed in Arrow and Fisher [1974], Henry [1974a,b]. These works first stressed the existence of an option value for irreversible investments, mainly in the context of environmental economics. Strangely enough, the term 'real options' was first coined by Myers [1977] in a problem of valuation of growth opportunities, nearly 10 years before McDonald and Siegel [1986]'s seminal paper on the option to invest. But the authors who certainly recognized the power of MacDonald and Siegel's approach to investment valuation were Avanash Dixit and Robert Pindick. Their book which reproduced most of their contribution to the field, has done a very important job in making the complex mathematical tools needed to perform real options analyses accessible to a large public. It contains the basic concepts and examples of real options but also presents more sophisticated models involving market equilibrium and even strategic behaviour.

3 Optimal investment rules

The point of view adopted here takes an alternative approach to expansion planning methods. Instead of developing large complex optimisation problems taking all the possible generation technology alternatives as well as all the uncertainties into account, it presents some optimal decision rules established in simplified situations using continuous-time finance methods and stochastic control theory. This approach, mainly developed under the real options flag, is at the origin of the progress on investment theory during the last quarter century. Historically speaking, the introduction of these two pillars in modern financial economic theory goes back before real options literature took off. The breakthrough by Black & Scholes in their 1973 paper on warrant pricing can be seen as one of the most striking results showing the power of continuous-time finance to formalize and solve dynamic economic models. Indeed, Paul Samuelson and Robert Merton can be considered the main promoters of continuous-time finance and stochastic control methods to obtain quantitative results in economic and financial modelling (Samuelson [1965], Merton [1992]). The models and results presented in this section can merely be considered the offspring of their vision and of this first breaktrough.

3.1 Uncertainty

Apparently the question of the effect of uncertainty on the intensity of investment seems worthy of no discussion. Indeed, from a firm's perspective, it looks obvious that decision-makers would be better off performing their investment in a less uncertain economy. Firms would tend to reduce their investment in times of great uncertainty to avoid suffering big losses from an inappropriate level of investment. If they invest less than what could they could have, they may regret missed profit. But if they invest too much, they may experience financial distress or even bankruptcy. In this line of thought, one would assert that uncertainty reduces the value of an investment.

When confronted with the economic literature, the intuition is no longer that clear. The question was addressed under so many different forms and contexts that it is not possible to present all the aspects of the controversy in this limited section. Indeed, the reader interested in this particular subject may find it useful to read Carruth et al. [2000]. The authors made an entire survey entitled "What do we know about investment under uncertainty?" summarizing more than 15 years of work on the subject and showing that the centre of gravity falls with the supporters of the intuition above. Here we will restrict our review to two models showing extremely divergent results. The first is Abel [1983]'s continuous-time model showing a positive relationship between uncertainty and investment and the second is McDonald and Siegel [1986]'s model showing an opposite relationship. Alternative models and empirical studies that provide insights on these theoretical results are also briefly reviewed.

Abel [1983] considers a risk-neutral, price-taking, profit maximizing firm with a homogeneous production function given by the Cobb-Douglas' production function $L^\alpha K^{1-\alpha}$ where L is the labour factor with a constant wage of w, K is the capital stock and $\alpha \in (0,1)$. The firm can invest I incurring an adjustment cost γI^β with $\beta > 1$. Its cash-flow at time t is then

$$\Pi_t = p_t L_t^\alpha K_t^{1-\alpha} - wL_t - \gamma I_t^\beta,$$

where p_t is the uncertain price of the output. The value of the firm is the expected present value of cash-flows:

$$V(K_t, p_t) = \max_{I_s, L_s} \mathbb{E}_t \left[\int_t^\infty e^{-(s-t)} \Pi_s ds \right],$$

The capital stock and the price follow the dynamics:

$$dK_t = (I_t - \delta K_t)dt \qquad dp_t = \sigma p_t dW_t,$$

where δ is the depreciation rate of the capital.

This problem is a standard stochastic control problem (see Pham [2011, chap. 3]). The value function is the solution of the HJB equation:

$$0 = rV - \sup_{I,L} \left[(I - \delta K)V_k + \frac{1}{2} p^2 \sigma^2 V_{pp} + pL^\alpha K^{1-\alpha} - wL - \gamma I^\beta \right]. \tag{10}$$

Solving for the supremum in I and L, one gets that the value function should satisfy the following PDE:

$$rV = -\delta K V_k + \frac{1}{2} p^2 \sigma^2 V_{pp} + hp^{1/(1-\alpha)} K + (\beta - 1)\gamma I^{\beta},$$

where $h = (1-\alpha)(\alpha/w)^{\alpha/(1-\alpha)}$. It turns out that the adjustment cost function was sufficiently well-chosen to allow an analytical solution.

The proposed explicit solution is:

$$V(K,p) = qK + \frac{(\beta-1)\gamma(\frac{q}{\beta\gamma})^{\frac{\beta}{\beta-1}}}{r - \dfrac{\beta(1-\alpha+\alpha\beta)\sigma^2}{2(1-\alpha)^2(\beta-1)^2}}, \qquad I = (\frac{q}{\beta\gamma})^{\frac{1}{\beta-1}},$$

$$q = \frac{hp^{\frac{1}{1-\alpha}}}{r + \delta - \dfrac{\alpha\sigma^2}{2(1-\alpha)^2}}.$$

The question of interest is whether or not an increase in uncertainty increases investment. In this model, the question boils down to a simple comparative statics problem of the variation of I with respect to the volatility of the price σ. As one can check on the solution, an increase in σ leads to an increase in I. The reason invoked for this counter-intuitive result is that as long as the marginal product of capital[7] is a convex function of the output price, the expected return of a marginal unit of capital rises with the price's volatility, making it more attractive to invest.

Abel [1983]'s result raised in-depth research on its robustness. In this model, the adjustment cost is symmetrical, making investments fully reversible: it is as costly to reduce the level of capital as to increase it. Moreover the firm is in perfect competition and risk-neutral. Issues regarding the adjustment cost function were first assessed. For instance, Abel [1984] showed that serial correlation of output prices can reverse his preceding result within the same framework. Pindyck [1988] showed that if the reversibility condition is suppressed, then uncertainty defers investment. Caballero [1991] showed that Abel [1983]'s result relies more on the convexity of the adjustment cost function than on its symmetry. Caballero's result was somewhat mitigated by Pindyck [1993], who pointed out a difference between firm-specific uncertainty and industry-wide uncertainty, showing that industry-wide uncertainty can have a negative effect on investment. Lastly, Abel et al. [1996] in a more general model show that uncertainty has an ambiguous effect on investment.

All the papers cited above deal with variations of Pindyck [1982] and Abel [1983]'s models involving production function and incremental investment. The model developed by McDonald and Siegel [1986] is based on a local approach and leads to an opposite conclusion. This model plays a particular place in the investment literature for several reasons. It precisely argues why the NPV rule is wrong in the case of irreversible investments and it shows that the NPV rule greatly differs from the real options rule. This result is important for the electric industry since it corresponds to its situation of irreversible capital-intensive investment. The authors' model (a perpetual American call option) make it possible to clearly see the impact of an increase in uncertainty for an industrial investor, even for a risk-averse investor.

[7] The short-term revenue is $\pi(K,L) = p_t L^{\alpha} K^{1-\alpha} - wL_t$, we have $\max_L \pi(K,L) = hp_t^{1/(1-\alpha)} K_t$ hence $hp_t^{1/(1-\alpha)}$ is the short-term marginal revenue of capital which expected value can be shown to be equal to q.

McDonald and Siegel [1986] consider a firm having the privately owned opportunity to invest at a cost I_t in an irreversible production asset whose present value is V_t. Their first argument is that since the investment is irreversible and that the decision to defer investment is reversible, a rational investor should pick up the best opportunity amongst all possible dates of investment. This rule is not new to capital budgeting theory: it is the decision rule for mutually exclusive projects. But here the exclusive alternatives apply to the same object (the production asset) at different times. Hence, as we have seen in section 2.3, the decision-maker's problem is:

$$L(V,I) = \sup_{\tau \geq 0} \mathbb{E}\left[e^{-\mu\tau}(V_\tau - I_\tau)\right], \tag{11}$$

where V and I are the initial values of V_t and I_t.

The present value V_t and the investment cost I_t are supposed to follow geometric Brownian motion $dI = \alpha_i I dt + \sigma_i I dW_i$, $dV = \alpha_v V dt + \sigma_v V dW_v$. The stochastic control problem is here an optimal stopping-time problem. The value function satisfies the variational inequality (Pham [2011, chap. 5]):

$$\min\left[\mu L - \mathcal{L}L, L - g\right] = 0$$

with $g(V,I) = V - I$ and

$$\mathcal{L}L = \alpha_v V L_v + \alpha_i I L_i + \frac{1}{2}\sigma_i^2 I^2 L_{ii} + \frac{1}{2}\sigma_v^2 V^2 L_{vv} + \sigma_v \sigma_i \rho V I L_{vi}$$

and where ρ is the correlation between the two Brownians. Noting that the problem is homogeneous in V and I and using smooth-paste conditions for the value function L, one is able to solve the preceding problem. The value function L is given by:

$$L(V,I) = \begin{cases} (c-1)I\left[\frac{V/I}{c}\right]^b & V \leq V^\star \\ V - I & V \geq V^\star \end{cases}$$

with

$$V^\star = \frac{b}{b-1}I, \qquad c = \frac{b}{b-1},$$

$$b = \frac{1}{2} - \frac{\alpha_v - \alpha_i}{\sigma^2} + \sqrt{\left(\frac{\alpha_v - \alpha_i}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\mu - \alpha_i)}{\sigma^2}}.$$

The solution exhibits a behaviour that can be easily described. In the region of (V,I) where V is lower than $V^\star$, nothing is done (continuation region) whereas in the other part of the plane, investment is made instantaneously (exercise region). When (V,I) touches the exercise frontier, investment is made. For standard parameter values (constant I, $\alpha_v = 2\%$, $\sigma_v = 20\%$, $\mu = 4\%$), Figure 2 (left) shows the exercise frontier as a function of V and I. The red dotted line represents the $V = I$ line. Note that the present value should exceed the investment cost by a factor greater than 2 for the investment to be undertaken. Moreover, it is easy to show that an increase in variance of V/I leads to an increase in the threshold value $b/(b-1)$, thus deferring investment.

This result is established here in the case of a risk neutral agent, or in a complete market setting. In this case, there is no ambiguity over the nature of the discount rate μ used in the problem (11). It is the risk-free rate. However, for investments in industrial projects, it is quite a strong hypothesis

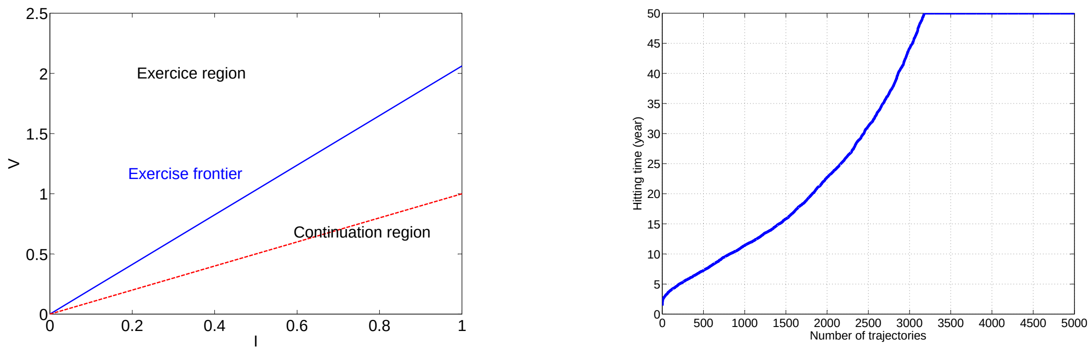

Figure 2: Exercise region and frontier for McDonald and Siegel [1986]'s investment model (left). Estimation of the repartition function of the first hitting-time for $V_t = V^\star$, with $\mu = 4\%$, $\sigma_v = 20\%$, $\alpha_v = 2\%$ with 5000 trajectories (right).

to consider that a project that is not even undertaken can be perfectly replicated by a portfolio of financial assets. For this more realistic situation, the discount rate μ is itself taken as function of the project's riskiness. Moreover, since financial assets cannot cannot perfectly replicate the project, its value depends on the decision-maker's risk preferences. McDonald and Siegel [1986] handle this case by determining the right risky discount-rate that should be used in relation (11). They show that the structure of the solution is preserved when changing α_i and α_f by $\delta_i = \hat{\alpha}_i - \alpha_i$ and $\delta_f = \hat{\alpha}_f - \alpha_f$ where $\hat{\alpha}_i$ (resp. $\hat{\alpha}_f$) is the expected return of a financial asset with a volatility equal to σ_i (resp. σ_f). The parameter δ_i behaves as an opportunity cost for deferring the investment in the asset: the greater it is, the costlier it is to hold the option and to defer investment. But the option to wait has still a positive value in this setting where market incompleteness is taken into account only by changing the discount rate. It is only recently that a treatment of the incomplete market case has been performed by taking the decision-maker's risk preferences the into account. Such an analysis has been developed successively in Henderson [2007], Hugonnier and Morellec [2007] and more recently by Grasselli [2011]. In a utility-based framework, the authors show that the time-value of the investment opportunity is still positive for an investment project that cannot be hedged by financial assets. Compared to the complete market case, the investment threshold is lower but as pointed out by Grasselli [2011], time flexibility still provides added value to an investment project even if it cannot be replicated.

Based on McDonald and Siegel [1986], many real option variants were developed amongst which the most popular are the option to abandon a project and the option to increase production capacity (He and Pindyck [1992]). As shown in section 2.3 the list of applications and variations is so abundant now that it makes no sense to systematically review them. But if many develop alternative stochastic control models that exhibit explicit solutions allowing comparative statics, few deal with empirical studies to test whether or not investors behave the way the real options rule recommends. As pointed out in Pindyck [1991], empirically testing the real options investment rule on aggregate data is quite an issue due to the very nonlinear behaviour of investment in this framework. Moel and Tufano [2002] avoid this difficulty by testing investors' real options forecast behaviour of in the case of gold mine management. The authors find that openings and closings of gold mines in the

US during the 1988-97 period followed patterns forecast by real options methodology (closing the mine when the gold price is well below the short-term production cost, and opening the mine when it is well above the short-term production cost).

We would like to conclude this section on the effect of uncertainty on investment decision timing by shedding some light on a natural concern of decision-makers. If there is an option to wait for investment, a natural question to ask is: how long can the decision-maker expect to wait before investing using the real options criterion? The law of the first-hitting time of a barrier above or below the initial condition of a geometric Brownian motion is perfectly known (see Jeanblanc et al. [2009, sec. 3.3]). Starting with a value V_0 below the threshold $V^\star$, it is known that V_t has a strictly positive probability to hit the threshold in finite time. In our particular case, and considering a constant investment cost I to simplify the discussion, if we denote the first hitting-time of $V^\star$ by $T^\star_{V^\star}$, its expectation is:

$$\mathbb{E}\left[T^\star_{V^\star}\right] = \begin{cases} \dfrac{1}{\alpha_v - \frac{1}{2}\sigma_v^2}\ln(V^\star/V_0) & \alpha_v > \frac{1}{2}\sigma_v^2, \\ +\infty & \alpha_v \leq \frac{1}{2}\sigma_v^2 \end{cases}$$

When the expected first-hitting time is finite, one recovers the effect of uncertainty on investment decision: an increase in σ_v leads to an increase in $\mathbb{E}\left[T^\star_{V^\star}\right]$, deferring investment. When it is infinite, comparative statics have no more meaning. But in the case when the expected first-hitting time is finite, one should note that the waiting period may be (very) long. For the realistic parameters $V_0 = I$, $\mu = 4\%$, $\sigma_v = 15\%$, $\alpha_v = 2\%$, the average time to wait before investing would be more than 80 years! Figure 2 (right) shows the results of the numerical simulation of the first hitting-time of $V^\star$ by V_t truncated to 50 years with the same numerical parameters and $I = 1$. In this case $V^\star = 2.01$. Figure 2 (right) shows that in 90% of cases, the decision-maker can wait more than seven years (of 250 days) before investing. In over 50% of cases, one could wait more than 30 years. During such a long time period, the decision-maker can be stuck in a very awkward situation. The NPV of the project can be very positive and nevertheless she should still refuse to reap the value of the project because it could be even higher in the future.

3.2 Time to build

McDonald and Siegel [1986]'s result put forward the hypothesis that the asset could be built instantaneously. In section 1.2, we saw that this is not the case for power plants where construction delays can reach 10 years for a nuclear power plant or a dam. This is also the case in many industries. However, Koeva [2000]'s study shows that the electricity sector exhibits both the longest construction delays and the greatest variance. Construction delays in electricity generation range from 12 to 225 months, whereas other sectors' construction delays range from six to 70 months. But does it really matter for an investment whose lifetime is expected to be more than 40 years? Are construction delays not small and negligible compared to lifetime expectancies? It seems it does matter at least at an aggregate level. Kydland and Prescott [1982] showed that the time to build is an essential feature of an equilibrium model to explain both the level of investment and its cycles. This result was confirmed by a second econometric study (Altug [1989]) and the relationship between investment cycles and time to build finds a nice mathematical foundation in Asea and Zak [1999]. The authors show that in the context of *deterministic* growth models of an economy with a single good, it is necessary to introduce time to build to get cycles.

Now, at a microeconomic level, what should be expected? Intuition suggests that investors should try to hurry to benefit from a favourable situation and to reduce the lost revenue of the building phase. But knowing that there are lost revenues during the building phase, decision-makers should wait for a higher price level or project value to compensate for these lost revenues. Thus, intuition provides the idea that the effect of time to build can be ambiguous or difficult to assess. The result obtained by Madj and Pindyck [1986] with an investment decision model in continuous-time with time to build follows the line of the intuition above. Taking an investment opportunity project whose value follows a geometric Brownian motion, they consider that the investment rate is bounded. Thus it takes several years to finish the project. Moreover, in their context the construction delay is both random and controlled: it depends on the evolution of the value of the completed project. The authors find that the time to build amplifies the negative relationship between uncertainty and timing. It induces a higher critical value triggering the investment (see Fig. 1, p. 21), thus deferring investment even more.

This result is based on a modelling of the construction delay where investors still have some flexibility. However, this is not the most common situation. Indeed, firms try to finish their building projects on time since not sticking to the schedule is perceived as bad project management. But modelling time to build as inflexible delays in stochastic control problems generally leads to infinite dimension problems (see Bensoussan et al. [2007]). Nevertheless, for the special dynamics of investment in production assets, it is often possible to reduce the problem to a finite dimension. This is exactly the case for the two models we are going to present here. Bar-Ilan and Strange [1996] and Bar-Ilan et al. [2002] are the main models that allow in-depth understanding of the effect of time to build on optimal investment rules. Bar-Ilan and Strange [1996] explicitly introduce a fixed time to build in Dixit [1989]'s investment problem for the optimal entry and exit of a project. The authors succeed in giving a quasi-explicit solution of the optimal decision rule. Bar-Ilan et al. [2002] extended this result to the case of an infinitely-lived representative agent maximising the social surplus. Their result is based on the solution of inventory problems in the early the 1960s in discrete time (Scarf [1959]) and extended to the case of continuous-time by the end of the 1970s (Constantinides and Richard [1978], Sulem [1986]). In their setting, the optimal investment rule is completely described although the threshold can only be computed numerically.

In Bar-Ilan and Strange [1996], a firm pays k to get an infinitely lived production facility of one unit of good per unit of time at a constant marginal cost of production w. It takes h units of time to build the facility. The investment cost k is paid at the end of the building phase and will be paid whatever happens during the building phase (irreversible decision). The firm can abandon its investment during the building phase at a cost l but reentry requires repaying the full cost k. The output price of production P is supposed to follow a GBM $dP = \mu P dt + \sigma P dW$ and cash-flows are discounted at a rate $\rho > \mu$. The firm faces two questions: when to initiate the project and when to abandon it? This problem can be formulated as an optimal switching problem. It leads to three variational inequalities instead of one as in problem (11). The exercise regions can be defined by two trigger prices P_H and P_L. When price P is higher than P_H, the firm switches from inactive to active (it initiates the project). When price P is lower than P_L, the firm switches from active to inactive (abandonment of the project). An important remark is that since the investment cost k cannot be recovered even if the project is abandoned, and because the abandonment cost l can be reduced by delaying, there is no economic advantage in abandoning during the building phase. Hence, abandonment will occur only at the end of the building phase.

Denote as $V_0(P)$ the value of the firm when it is inactive, $V_1(P)$, the value of the firm when it

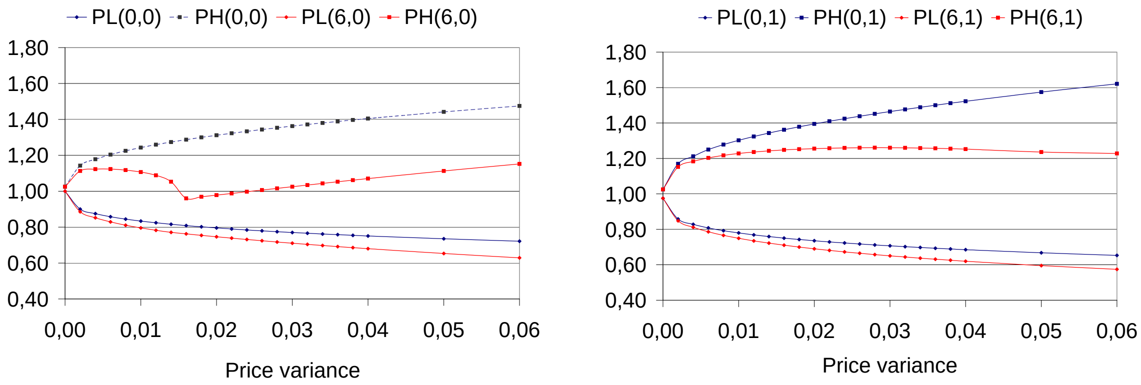

Figure 3: Bar-Ilan and Strange [1996] trigger prices $P_L(h,l)$ and $P_L(h,l)$ for $\rho = 2.5\%$, $\mu = 0$, $w = 1$, and $k = 1$ without abandonment cost (left), with an abandonment cost of $l = 1$ (right).

is active and the project is completed and $V_2(P,\theta)$ the value of the firm when it is active but the project is not completed yet, with $0 \leq \theta \leq h$ the remaining time before completion. The PDEs satisfied by the value functions V_0 and V_1 are standard application of stochastic control framework:

$$\rho V_0 - \mu P V_0' - \frac{1}{2}\sigma^2 P^2 V_0'' = 0, \tag{12}$$

$$\rho V_1 - \mu P V_1' - \frac{1}{2}\sigma^2 P^2 V_1'' - P + w = 0 \tag{13}$$

However, to establish the PDEs satisfied by V_2, one should notice that the discounted value of the firm should not change during the building phase because it is always better to pay the abandonment cost later and because the investment cost is sunk and paid only at the end of the building phase. Thus:

$$\mathbb{E}\left[e^{-\rho dt} V_2(P(t+dt), \theta - dt)\right] = V_2(P(t), \theta)).$$

And using Ito's lemma, one finds:

$$\rho V_2 - \mu P \frac{\partial V_2}{\partial P} - \frac{1}{2}\sigma^2 P^2 \frac{\partial V_2}{\partial P^2} + \frac{\partial V_2}{\partial \theta} = 0 \tag{14}$$

Moreover, the connection between the different parts of the project value is made noting that near completion, the firm should either abandon or keep the project. Hence,

$$V_2(P,0) = \begin{cases} V_1(P), & P \geq P_L, \\ V_0(P) - l & P \leq P_L \end{cases}.$$

It is not possible to provide an analytical solution for this system of ODEs and PDEs. The analysis is reduced to special limiting cases and numerical illustrations. We provide here the main result of this model. Figure 3 (left) shows the effect of an increase in price volatility for two cases of time to build (no time to build and a six-year delay) when there is no abandonment cost. Although the abandonment threshold clearly decreases as volatility increases irrespective of the construction delay, this is not the case for the investment threshold. Figure 3 (left) shows a very nonlinear behaviour of the investment threshold. One notices that there is a range of volatility where the investment threshold can decrease with an increase of volatility. For this range of volatility, the investment threshold reaches a local minimum that is even lower than the certainty trigger. In this case, the certainty trigger is $w + \rho k = 1.025$. The explanation of these counter-intuitive results lies in the effect of the abandonment cost. Figure 3 (right) shows that the introduction of an abandonment cost drastically reduces the former behaviour. When there is an abandonment cost, must substantially increase volatility to be able to observe a slight decrease in the investment threshold. The comparison of the investment threshold with and without abandonment cost reveals the cause of the uncertainty-investment positive relationship in the case of long delays. The ability to abandon the project at no cost makes it possible to truncate the eventuality of low profit. Since the investment cost is paid at the end of the building phase, the decision-maker can benefit from situations where the bad news of future low profit is learned during that period. The decision to abandon the project will be taken immediately but the decision-maker will earn the time-value of the investment cost. In some situations, this benefit can outweigh the opportunity cost of waiting.

Even though this result is striking, it should be noted that it also has a limited impact on the standard result on the relationship between uncertainty and investment. The result is obtained for a certain range of parameters, meaning that situations where long delays hasten investment irrespective of uncertainty can occur, but that is not the general case in this model. It also heavily relies upon the hypothesis that the investment cost is paid at the end of the building phase. In Bar-Ilan et al. [2002], the authors study the joint effect of uncertainty and time to build in an equilibrium model which provides a more robust setting. The problem of interest here is to meet the demand for electricity at a minimal cost. There is only one available technology and it takes h years to build. Linear penalization is incurred in both situations of excess or lack of capacity. Formally, the problem is written as an impulse control problem.

The firm cannot adjust demand to production by pricing signal (no demand-response management). Building a new production capacity takes time h. The installation cost is

$$C(\xi) = \begin{cases} 0, & \xi = 0 \\ k + c\xi, & \xi > 0. \end{cases}$$

Once installed, a capacity lasts forever. Excess capacity is the difference between existing production capacity and current demand and is noted y. Holding an excess or a lack of capacity makes the firm incur a cost $f(y)$ such that:

$$f(y) = \begin{cases} -py, & y \leq 0 \\ qy, & y > 0 \end{cases}$$

There is no possibility to remove capacity. With $h = 0$, the system is well described at time $t = 0$ by the excess capacity x. With $h > 0$ the description of the system requires to remember *all* investment decisions ξ_i and their corresponding time τ_i for $i = 1, ..., n$. The state of the system is given by the vector (x, Ψ) with $\Psi = \{(\tau_i, \xi_i)_i\}$ and $-h < \tau_1 < ... < \tau_n < 0$. The variable $y_{(x,\Psi)}$ denotes the excess capacity at time $t \geq 0$ when the state was (x, Ψ) at time $t = 0$. The dynamics of y is given by:

$$dy_{(x,\Psi)} = -gdt + \sigma dW_t + \sum_{(\tau_i,\xi_i)\in\Psi} \xi_i \mathbb{I}_{t-\tau_i-h} + \sum_{i\geq 1} \eta_i \mathbb{I}_{t-\theta_i-h}$$

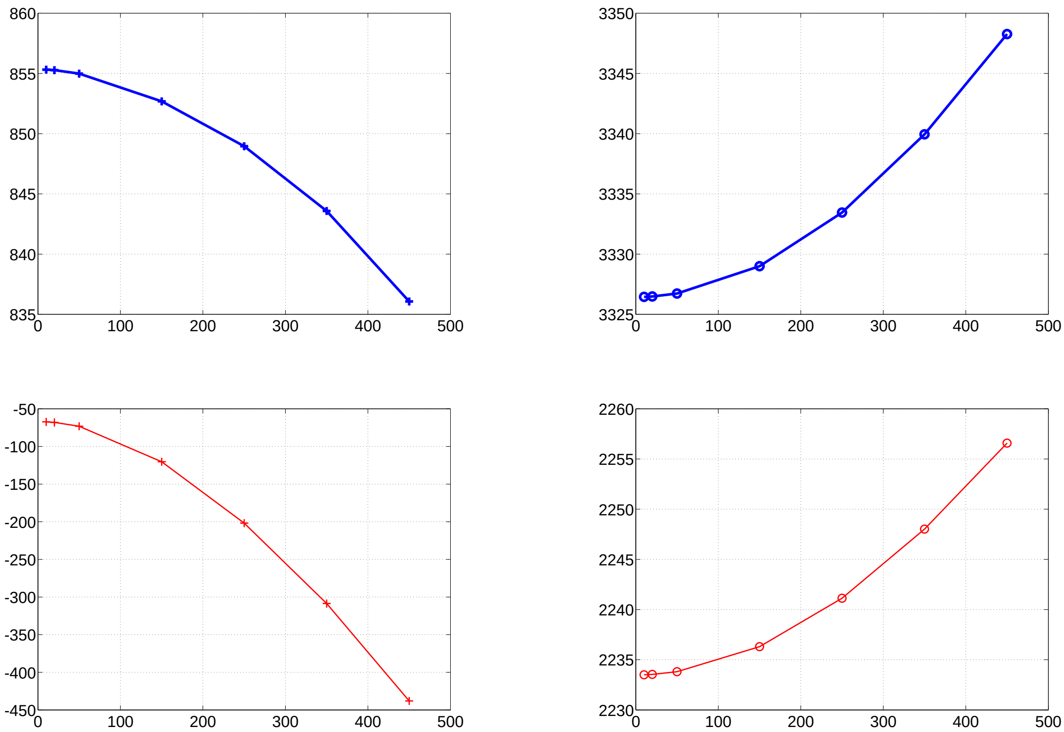

Figure 4: Bar-Ilan et al. [2002] — target (above) and trigger (below) variation w.r.t. σ for a 1-year time to build (left) and for an 8-year time to build (right).

with the initial condition $y_{(x,\Psi)}(0) = x$ and where (θ_i, η_i) denote the investments done after $t = 0$. Note that the electricity demand is modelled as an arithmetic Brownian motion. Lastly, $U(x, \Psi)$ denotes the minimum expected cost reached by the optimal investment strategy when the firm was in the state (x, Ψ) at time zero.

$$U(x, \Psi) = \inf_{\theta_i, \eta_i} \mathbb{E} \left[\int_0^\infty e^{-\alpha t} f(y_{(x,\Psi)(t)}) dt + \sum_{i \geq 1} e^{-\alpha \theta_i} C(\eta_i) \right].$$

The first important point to note is that the optimal control (θ_i, η_i) is a function of *only* the *committed capacity* $x + \sum_{1 \leq i \leq n} \xi_i$. The optimal solution does not depend on the timing of past investment decisions but only on their total amount. It is only necessary to remember their sum to take decisions. This point is linked to the linear dynamic of investment. This remark is extensively used by other studies (Aguerrevere [2003], Grenadier [2000, 2002]). Using this remark and previous results on inventory management (Scarf [1959], Constantinides and Richard [1978], Sulem [1986]), the authors prove that the optimal solution satisfies a trigger/target form. When the committed excess capacity (difference between the committed capacity and the demand) reaches a level s, it is optimal to invest exactly to reach the new level S of committed excess capacity. The trigger s and target S are explicitly given by a nonlinear system that is solved numerically.

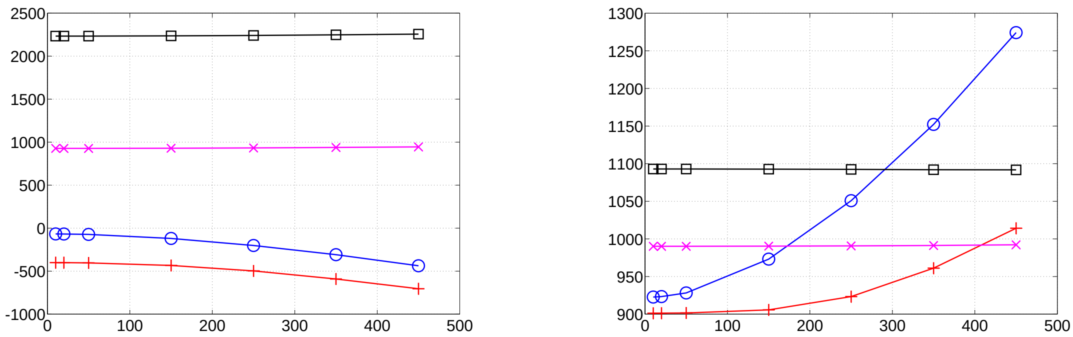

Figure 5: Bar-Ilan et al. [2002] — Variation of the investment trigger for no delay (red plus), a one-year delay (blue circle), a four-year delay (magenta cross) and an eight-year delay (black squares) (Left). Variation of the invested quantity for no delay (red plus), a one-year delay (blue circle), a four-year delay (magenta cross) and an eight-year delay (black squares).

The main result of Bar-Ilan et al. [2002] relies on the numerical illustration of the behaviour of the trigger and target values when time to build and uncertainty are increased. A reference situation is provided with parameter values: $p = 250$ \$/kW per year, $q = 100$ \$/kW per year, $k = \$100$ million, $c = 1000$ \$/kW, $g = 350$ MW/year, $\sigma = 250$ MW/year, $\alpha = 5\%$.

Figure 4 plots the variations of the investment triggers and targets as a function of demand uncertainty σ for two different construction delays, one and eight years. First, one notices that variations are reversed for long delays. The trigger increases with the uncertainty for an eight-years delay whereas it decreases for a one-year delay. The same holds for the investment target. Hence in the case of a long construction delay, the investor will invest sooner. Bar-Ilan et al. [2002] can be completed by pointing out that this effect is nevertheless very small. Indeed, in the case of an eight-year delay the curves are rather flat. In fact, with a long construction delay, the investment triggers and targets become insensitive to uncertainty. Figure 5 illustrates this point. On the left, we see the monotonic relation between investment *timing*, uncertainty and the time to build measured by the investment trigger. The longer the delay, the sooner the investment will be made since the excess committed capacity increases monotonically with h. But triggers for small delays exhibit a non-negligible sensitivity to uncertainty whereas this is no longer the case for long delays. This result is even more striking on the *invested quantity* as measured by $S - s$. The one-year delay curve presents a high sensitivity to uncertainty that disappears for the four and eight-year delay cases. In a one-year delay case, for high levels of uncertainty, the invested quantity will be even greater than in the eight-year delay case. Hence, with small delay and high uncertainty, one invests less often than with long delays but with a greater intensity.

This insensitivity of investment to uncertainty in the case of long time to build does not mean that uncertainty has no effect on the investment dynamics in this case. In fact, the main effect of a longer time to build is to compel the system to live with a high amount of committed excess capacity. Figure 4 (below, left) shows that for a short construction delay, the system can allow itself to spend some time with a negative committed excess capacity and it can handle the equilibrium with no more than a committed excess capacity of 850 MW. With a longer construction delay, this is

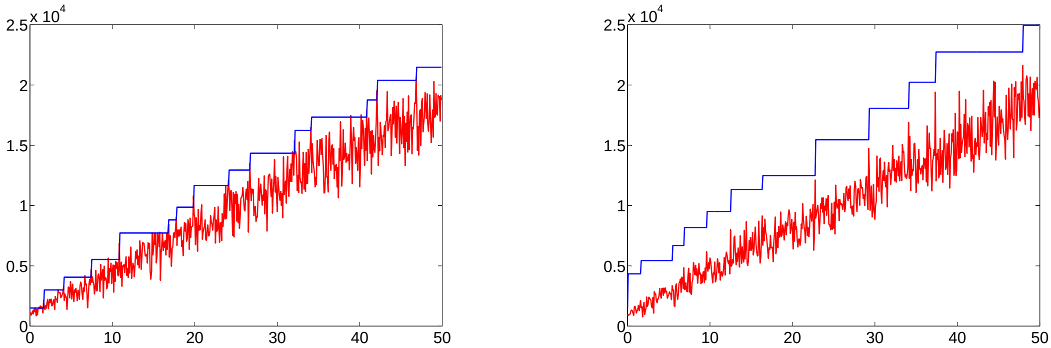

Figure 6: Bar-Ilan et al. [2002] — Two examples of committed capacity behaviour for a one-year delay (left) and an eight-year delay (right)

no longer possible. The decision-maker will be compelled to invest as soon as the committed excess capacity falls under a very positive value of 2,200 MW. It means that the system is constrained to maintain a large amount of installed capacity over the actual demand. This point is illustrated in Figure 6, where the reader can compare the investment dynamics in the case of a one-year and a eight-year delay. In a way, Bar-Ilan et al. [2002]'s model shows that a long construction delay makes uncertainty a second-order problem, since the optimal response in that situation is to maintain a wide capacity margin. Indeed, this fact is a basis of power system reliability.

3.3 Competition

Although very intuitive, the idea that competitive pressure would erode and make the time value of the investment option leads disappear to difficult mathematical modelling problems. It may appear for some as the simple application of competitive equilibrium principles to investment timing and therefore should not deserve more than a simple footnote in an economic paper as Marglin [1967] did. Competition drives prices to marginal costs and profits to zero. But developing a continuous-time stochastic model to recover this intuition and quantify the speed of this erosion raises considerable difficulties. The first mathematical models used to quantitatively assess this question were either stochastic but in discrete time model or deterministic but in continuous-time. Moreover, their findings were surprisingly not necessarily a confirmation of the intuition. In a deterministic continuous-time framework, Fudenberg and Tirole [1985a] show that the possibility of preemption leads to rent equalization in the case of duopoly but not in oligopoly with more than three firms. In a continuous-time stochastic control model context, Leahy [1993] showed a very surprising result. The optimal *timing* of investment is independent of the competition pressure. Leahy [1993] considers a continuous-time investment model where irreversible investments can be continuously incremented by a set of identical firms. One would expect that firm's optimal timing would depend on the level of investment and on the strategy of competitors. The author shows that it is enough for a firm to assume that the investment level will remain constant *as if* competitors were not investing. A pure *myopic* behaviour provides the correct timing. This result goes against the idea that competition hastens investment. Here the investment timing is not impacted by competition.

Only the invested quantities are. The intuition behind this phenomenon is that in both cases, the investment threshold is the same. It corresponds to a price level making the investment cost and the expected profit equal. These counter-intuitive results obtained in continuous-time contrast with the Spatt and Sterbenz [1985] discrete-time stochastic model, where the intuitive result is obtained. As the number of rivals increases, the investment rule tends to the NPV rule.

The development of an equilibrium model taking investment and competition into account requires being able to define and compute a Cournot-Nash equilibrium in a stochastic control context. To our knowledge, Grenadier [2002]'s work which is presented here is the first paper to provide an analytical solution of a complete equilibrium model of investment under uncertainty allowing an in-depth quantification of the competitive pressure on the investment option. Grenadier [2002]'s model and resolution method are based on several previous breakthroughs. It uses Leahy [1993]'s prior result to reduce the difficulty of computing the Nash equilibrium. Moreover it takes many of the different lines of Williams [1993], Baldursson [1998], Kulatilaka and Perotti [1998] who developed similar models.

Grenadier [2002] considers an oligopolistic industry composed of n identical firms producing the same homogeneous good. The variable $q_i(t)$ represents the production of firm i at time t. Production of each firm is supposed to equal its capacity. There is no possibility to decrease capacity. Define the total production $Q(t) = \sum_i q_i(t)$ and the total production of firms other than j $Q_{-j}(t) = \sum_{i \neq j} q_i(t)$. The endogenous price process is given by

$$P(t) = D(X(t), Q(t))$$

with D the inverse demand function and X an exogenous shock process affecting the demand. The demand function D is supposed to be regular enough, increasing in X and decreasing in Q. The demand shock X is supposed to follow a diffusion process

$$dX = \mu(X)dt + \sigma(X)dW.$$

There is no variable cost of production. The instantaneous profit function reads

$$\pi_i(X, q_i, Q_{-i}) = q_i D(X, Q_{-i} + q_i)$$

for firm i when producing q_i. At each time t each firm can invest continuously with a linear cost K per unit. Firms are assumed to be risk-neutral with r denoting the risk-free rate.

The production processes (q_i^*) form a Nash equilibrium if q_i^* is an optimal strategy for firm i when it takes the strategies of its competitors Q_{-i}^* as given. Denote by $V^i(X, q_i, Q_{-i}; q_i(t), Q_{-i}(t))$ the value of firm i for strategies $q_i(t), Q_{-i}(t)$ with (X, q_i, Q_{-i}) as an initial condition. One has

$$V^i(X, q_i, Q_{-i}; q_i(t), Q_{-i}(t)) = \mathbb{E}\left[\int_0^\infty e^{-rt}\pi_i(X(t), q_i(t), Q_{-i}(t))dt - \int_0^\infty e^{-rt} K dq_i(t)\right]$$

And the controls (q_i^*) form a Nash equilibrium, meaning that for all i,

$$V^i(X, q_i, Q_{-i}; q_i^*(t), Q_{-i}^*(t)) = \sup_{q_i(t)} V^i(X, q_i, Q_{-i}; q_i(t), Q_{-i}^*(t)).$$

At this point, it is necessary to make a comment on the definition of a Nash equilibrium in this context. Back and Paulsen [2009] raised the issue that this definition is only suitable for *open-loop* strategies as opposed to a more general possible set of strategies which are *closed-loop* strategies. In the above definition of equilibrium, strategies are defined as *commitments*. Along the optimal trajectories, firm i's response is well-defined but if any player deviates from the equilibrium, firm i will still keep on investing as if it was an optimal response. In this particular setting, Back and Paulsen [2009] exhibit an open-loop strategy allowing a better payoff than the closed-loop equilibrium strategy, showing that Grenadier's set of equilibrium strategies is somehow too small. Nevertheless, since Back and Paulsen [2009] admit that defining a closed-loop equilibrium in this context is still an issue, we will stick to Grenadier's model, keeping in mind that it is limited to commitment strategies. We will see below that this restriction on the possible strategies of the firms has an impact on the investment option value.

The author focuses on a symmetrical Nash equilibrium. All firms have access to the same rights, technology and information, so that $q_i^* = q_j^*$ for all i, j and $q_i^* = Q^*/n$. This hypothesis greatly simplifies the equilibrium equations, bringing the dimension of the problem from an $n+1$ to 2. Moreover, it is assumed that the optimal strategy of firm i is given as a threshold $X^i(q_i, Q_{-i})$. Using the fact that all firms are identical and that only symmetrical equilibrium is sought, all firms have the same exercise threshold $\overline{X}(q_i, Q_{-i})$. Moreover, applying Leahy [1993]'s method in this context helps to show that this threshold is equal to $X^m(q_i, Q_{-i})$, the investment threshold of a myopic firm, i.e. making the assumptions that $Q_{-i}(t) \equiv Q_{-i}$. Thus the investment threshold boils down to a function of only the aggregate output Q, that is denoted $X^*(Q)$. Let $M^i(X, q_i, Q_{-i})$ denote the value of the myopic firm i that considers that the current level of supply by competitors Q_{-i} will remain constant, and its marginal value

$$m(X,Q) = \frac{\partial M^i}{\partial q_i}(X, \frac{1}{n}Q, \frac{n-1}{n}Q).$$

The functions m and X^m are determined by the following PDE:

$$rm - \mu(X)m_X - \frac{1}{2}\sigma^2(X)m_{XX} - D(X,Q) - \frac{Q}{n}D_Q(X,Q) = 0,$$

with the boundary conditions:

$$m(X^*(Q), Q) = K, \qquad m_X(X^*(Q), Q) = 0.$$

Here it should be stressed that the initial system of PDEs satisfied by the value of each firm is amazingly reduced to a single PDE with a one dimensional exercise frontier. This result allows the computation of the explicit solution in special cases of inverse demand function and demand shock. The main case considered is an inverse demand function given by $P(t) = X(t)Q(t)^{-1/\gamma}$ with $\gamma > 1$ and a geometric Brownian motion $dX = \mu X dt + \sigma X dW$ for the demand shock. In this case, the marginal value of a myopic firm is

$$m(X,Q) = -\frac{n\gamma - 1}{n\gamma}\frac{v_n^{1-\beta}}{\beta(r-\mu)}Q^{-\frac{\beta}{\gamma}}X^\beta + \frac{n\gamma - 1}{n\gamma}\frac{Q^{-1/\gamma}}{r-\mu}X,$$

the investment threshold is

$$X^*(Q) = v_n Q^{1/\gamma},$$

and the optimal aggregated investment policy:

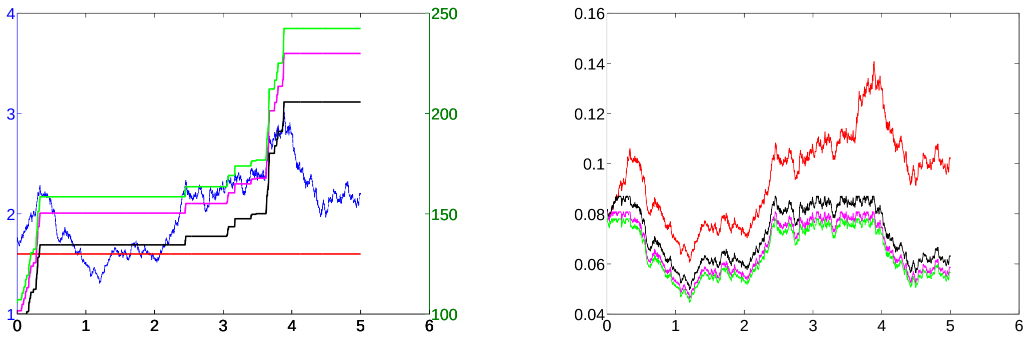

Figure 7: Grenadier [2002] — Investment trajectory for $n = 1, 5, 10$ and 20 firms for a given demand shock trajectory (left). Price trajectories for the same demand shock trajectory and the same number of firms (right).

$$Q(t) = \max\left[Q(0), (\frac{Y(t)}{v_n})^{\gamma}\right],$$

with

$$v_n = \frac{\beta}{\beta-1}\frac{n\gamma}{n\gamma-1}(r-\mu)K, \qquad Y(t) = \sup_{s\le t} X(s),$$

$$\beta = \frac{-\mu + \sigma^2/2 + \sqrt{(\mu-\sigma^2/2)^2 + 2r\sigma^2}}{\sigma^2}.$$

From these relationships, the behaviour of the investment threshold when the number of firms increases is straightforward since it is expressed directly in the parameter v_n. It is a decreasing function of n. The more competitors, the sooner investment is made. The intuitive effect of competition on the timing of investment is recovered here. With fierce competition, an investment opportunity is seized sooner. This result is illustrated in Figure 7 (left), where optimal investment trajectories are plotted for the same demand shock trajectory of X. The monopoly (red curve at the bottom of the curve) does not invest at all whereas it is enough to have five firms (black curve) to see an increasing investment following the demand shock. Competition not only pushes firms to invest sooner, but they also invest more as shown by the situations with 20, 10 and five firms. Thus the downward effect of competition on the price is not surprising. Figure 7 (right) displays a nice ordering of the price trajectories where the monopoly situation lies on top and the 20 firms case at the bottom.

Now let us analyze the option value of an investment. Let $G(X, Q)$ denote the value of the Qth unit of investment in production providing the perpetual cash-flow $P(t) = D(X(t), Q(t))$. The difference $G(X^*(Q), Q) - K$ represents the NPV of the incremental unit of investment done in an optimal way. The ratio $\Lambda(n) = (G(X^*(Q), Q) - K)/K$ represents the option premium of this incremental investment. It is shown that $\Lambda(n)$ takes the following simple form

$$\Lambda(n) = \frac{1}{n\gamma - 1}.$$

As the number of competitors tends towards infinity (perfect competition), the investment option value tends to zero and the NPV rule is recovered. It seems that the correct intuition that competitive pressure erodes the time value of an investment option is recovered. But the result above is not fully what would have been expected. For a small number of identical firms, there is still a non-null value over the net present value. It would have been expected that even with two equivalent firms, preemption would lead to a complete erosion of the investment option value since investment can be done continuously in time. We will see in the next section that even in a symmetrical duopoly facing a single investment opportunity, it is not easy to recover this intuitive result. But here the problem lies in the difficulties of properly defining continuous-time stochastic games with continuous and unbounded decision variables. Recent remarks by Back and Paulsen [2009] point to precisely this problem. Equilibrium in Grenadier [2002]'s model refers to a set of strategies that excludes the full possible range of competitors' responses. Nevertheless, the difficulties involved with continuous-time stochastic games did not prevent further developments along Grenadier [2002]'s lines (see Aguerrevere [2003] for a variant with flexible production and Novy-Marx [2007] for an impulse control variant).

3.4 Strategic interactions

The main driver of the competitive situation studied in the preceding section was the fear that other firms would make the investment first. Nevertheless, not all competitive investment situations take the form of a fear of being preempted. There are cases where it is better to be the second one to invest. This is the case for instance of R&D investment. The first one may spend a many resources on finding an innovation that competitors can duplicate at low cost by using reverse engineering once it is in the market. This is also the case for offshore exploration leases. Oil companies buy some leases to make offshore exploration for a limited period of time (less than 10 years). The tracts where explorations are to be made are close one to another. Thus a company's drilling whether successfull or not informs the competitor on the probability of success. Here both have an interest to wait and be the second so as to benefit from this information spillover. These situations are referred to as *war of attrition* and the example is taken from Dias [1997].

Fudenberg and Tirole [1985b,a] did provide important material to assess strategic interactions of investment decisions in a deterministic game theory context. Soon after the first papers on real options, economists realized that continuous-time finance methods could also help to give insights into strategic competitive situations under *uncertainty*. This led to the development of a field of its own under the label of *option games*. It is a very active field from both applied mathematics and economics viewpoints. The economic analysis of all kinds of oligopolistic situations is being currently investigated. Interested readers can refer to an increasing number of monographs amongst which Smit and Trigeorgis [2004] hold a leading role, while Huisman [2001] was the first book published on the subject and Thijssen [2004] is contemporary. For a shorter introduction, one can read Dias and Teixeira [2010]'s review of the subject, which covers the historical development of the field, the main economic results obtained in the literature on option games and the tracks for potential future mathematical methods that would help to allow the same level of computational flexibility in multi-actor situations as well as in a single-actor situation.

Here we will limit ourselves to Smets [1993]'s seminal work on irreversible investment in duopoly. Although never formally published in academic journals, it has led to many developments (Baldursson [1998], Kulatilaka and Perotti [1998], Lambrecht and Perraudin [2003]) and is reproduced in

Dixit and Pindyck [1994, chap. 9].

Consider a one-shot investment situation where two firms each have the potential to introduce one unit of production capacity at the same investment cost I. Both firms know they are the only two firms that can perform this investment. This corresponds to capitalistic industries where a limited number of firms have the financial and technical resources to perform the investment. Once installed, the new capacity is supposed to be used at its maximum level. The inverse demand function for the produced good is $P = YD(Q)$ where Q is the production level. It can take three values (0, 1 or 2) leading to three different demand values $D(0)$, $D(1)$ and $D(2)$, ranked in decreasing order. The initial demand is $D(0)$. The variable Y is the demand shock supposed to follow a geometric Brownian motion $dY = \alpha Y dt + \sigma Y dW$. Thus if one firm invests, the demand will decrease to $D(1)$ and the price will be negatively impacted. And if both firms invest, this effect will be amplified and the demand will drop to $D(2)$. So the question is what is the optimal investment rule for both players. The response will also help to answer the question of whether or not the fear of preemption will destroy the time value of the investment option.

The problem is a continuous-time option game with a finite set of possible actions. This point substantially simplifies its resolution. To focus only on the effect of competitive pressure, both firms are supposed to be risk-neutral and the risk-free rate will be denoted r. The problem is solved backwards. We suppose that one firm has already invested and we look at the optimal response of the second firm. Knowing the optimal response of the second firm to invest, we look at the optimal decision of the first firm. Two possible situations can be studied: symmetrical situation or pre-assigned leadership. In the first situation, both firms can invest first. In the second, one firm is designated to invest first (the Leader) and the second one to invest after (the Follower).

Consider first the symmetrical situation. The follower's profit will be $YD(2)$. As we have learned from previous examples, the follower invests when the demand shock Y will reach a certain threshold Y_2 to be determined. Following the same method as in section 3.1, it can be found that:

$$Y_2 D(2) = \frac{\beta_1}{\beta_1 - 1}(r - \alpha)I$$

with

$$\beta_1 = 1/2 - \alpha/\sigma^2 + \sqrt{(1/2 - \alpha/\sigma^2)^2 + 2r/\sigma^2}.$$

If $Y_2 \leq Y$, the follower invests immediately and gets

$$\frac{Y_2 D(2)}{r - \alpha} - I.$$

If $Y \leq Y_2$, the follower waits until the first time Y reaches Y_2 and then gets $\frac{Y_2 D(2)}{r-\alpha} - I$. Its expected present value is then

$$\mathbb{E}\left[e^{-r\tau}\right]\left(\frac{Y_2 D(2)}{r - \alpha} - I\right)$$

with

$$\tau = \inf\left\{t \mid Y_t = Y_2\right\}.$$

This computation can be done explicitly and the value of the follower is obtained as:

$$V_2(Y) = \begin{cases} YD(2)/(r - \alpha) - I & \text{if } \; Y_2 \leq Y \\ \\ (Y/Y_2)^{\beta_1}\left[YD(2)/(r - \alpha) - I\right] & \text{if } \; Y \leq Y_2 \end{cases}$$

Now that the value of being a follower is known, let us determine the optimal investment strategy and the value of the leader. The leader knows that if Y is below Y_2, the follower will wait until Y hits Y_2. Thus as long as $Y < Y_2$, if the leader invests, it will collect the profit $YD(1)$. Hence, its expected present value is

$$\mathbb{E}\left[\int_0^\tau e^{-rt} YD(1)dt\right] - I + \mathbb{E}\left[e^{-r\tau}\right] \frac{Y_2 D(2)}{r-\alpha},$$

with τ representing the same hitting time as above. It is composed of two parts: The profit from investing at time zero and being alone until τ and the profit from being two in the market since τ. This expectation can be explicitly computed and the leader's value function can be deduced:

$$V_1(Y) = \begin{cases} V_2(Y) & \text{if} \quad Y_2 \le Y \\ \frac{YD(1)}{r-\alpha}\left[1 - (Y/Y_2)^{\beta_1 - 1}\right] & \\ \quad +(Y/Y_2)^{\beta_1} \frac{YD(2)}{r-\alpha} - I & \text{if} \quad Y \le Y_2 \end{cases}$$

The value functions of both the leader (V_1) and the follower ($V2$) are represented in figure 8 (left). Two remarks can be made here. First, even in a symmetrical situation, it is not always better to be the leader, i.e. to invest first. There is a threshold Y_1 under which the cost incurred by investing first is not covered by the flow of profit. Thus, if the situation starts with Y below Y_1, neither firm will invest since it is better to be the follower. But as soon as Y exceeds Y_1, it is better to be the leader. In the symmetrical case, both firms will invest and then neither receives the excess flow of profit of being alone in the market. They both receive only $(Y_1 D(2))/(r-\alpha) - I$, which is less than the follower's value. Second, suppose now that the leader is randomly chosen or that one firm reacts quicker than the other and becomes the leader. At the threshold Y_1, one can check that the flow of profit exceeds the investment cost $I \le Y_1 D(1)/(r-\alpha)$. Knowing that the follower's future investment will reduce the flow of profit, the leader's intention is to invest only at the threshold that compensates for this future loss. The first investment is made with a positive net present value and not a null net present value. Thus competition does urge firms to invest since both of them would like to invest at the same time. But it does not lead to a null net present value investment threshold.

The fact that there is no procedure to determine a leader and a follower leads to some kind of paradoxical situation. What occurs does not correspond to what was computed by both rational agents. The first investment is expected to have an excess return over the investment cost but this is not likely to happen since the system is going to jump immediately from $D(0)$ to $D(2)$ instead of $D(1)$ as expected. Indeed, one needs a way to determine in advance who is the leader and who is the follower so that both players can use this information in their computation.

We are going to see that things are quite different if the roles have been pre-assigned. Now, the leader has the ability to wait because there is no more preemption threat. The leader faces the following problem to solve:

$$\sup_{\tau_1} \mathbb{E}\left[\int_{\tau_1}^\tau e^{-rt} YD(1)dt\right] + \mathbb{E}\left[e^{-r\tau}\right] Y_2 D(2)/(r-\alpha) - I$$

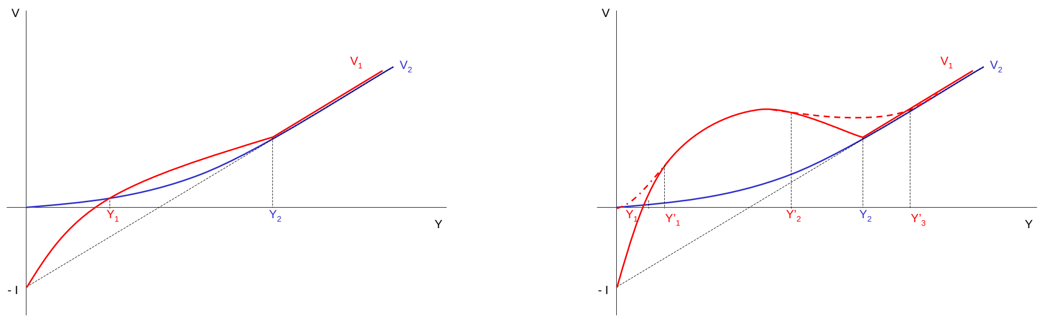

Figure 8: Value functions for a leader-follower one-shot investment game in a symmetrical case (left) and in a pre-assigned case (right). Leader's value function V_1, follower's value function V_2

where τ is still the first hitting time when $Y = Y_2$. We skip here the details of the resolution of this new problem to focus on the qualitative results of this model. The new value functions are represented in figure 8 (right). Now, the investment region of the leader is no longer connected. The leader invests either if $Y \in [Y_1', Y_2']$ or if $Y \geq Y_3'$. The first interval corresponds to an investment situation where the price is high enough to justify the investment ($Y \geq Y_1'$) but low enough to enjoy enough time being alone in the market since the leader knows that his competitor is waiting for the price to reach the threshold $Y_2 D(2)$ to make his move. The second part of the investment region corresponds to an unusual situation. The leader's investment threshold is above the follower's, meaning that the follower would invest if he were not compelled to wait for the leader's move. But knowing that as soon as the leader invested, the follower would do the same, the leader is waiting for the price to reach an excess value to compensate from the immediate loss that the leader would incur due to the follower's move.

By this last example, we hope to have shown the very non-intuitive results that can occur when investment involves a strategic dimension.

4 Conclusions

We have tried to show that continuous-time finance methods and stochastic control theory can provide in-depth quantitative analysis of optimal investment strategies. We have seen that they provide tractable models to assess the effects of both uncertainty and construction delays. They also make it possible to offer quantitative measures of the effect of competitive pressure as well as a nice setting to analyze strategic investment in the case of duopolies. But it is not possible to conceal that the progress made by the theory of investment under uncertainty during this last quarter century barely translated into firms' operational capital budgeting processes. So one should ask first if firms have to change their investment methods and what work should be done to help fill the gap between theoretical recommendations and methods used in practice.

4.1 What investment rule should be applied?

Does all this methodological progress translate into new investment rules for decision-makers in the electricity sector? Power producers are facing competitive pressure on investment for production assets with common access, and thus the simple NPV rule with all its known shortcomings should be enough for them. But there are situations where power utilities could get some insights by using the approaches presented in this review. For instance, many power utilities have a monopoly on the network (transport or distribution). Thus, they are in a situation where their investment decision should be made without neglecting their time-value. Moreover, power utilities are facing wars of attrition on their retail markets. The first to increase its price because of an increase in gross market price generally incurs a higher market share loss than the followers. So there is a trade-off to be found between a sure money loss because of an increasing supply cost and an uncertain future loss caused by a decreasing market share.

But the methods presented here are still complex and require highly skilled mathematicians to be applied. They have not led to simple recipes that can be mechanically applied. One cannot expect a decision-maker to embrace altogether the complexity of industrial projects, the principles of the financial process and the subtleties of singular control theory or impulse control theory. Now even the real options trademark is being counterproductive in companies as it may appear to managers as a sophisticated tool that provides non-intuitive results based on unrealistic assumptions.

4.2 Research prospects

These observations prompt us to propose some guidelines for further research to help fill the gap between theoretical methods and their applied counterparts.

First, although very interesting from an economic perspective, the comparative statics studies on the effect of uncertainty modelled as a Gaussian noise on investment timing is of less interest than developing analyses with more realistic price modelling. It is possible to do it. Research papers already exist presenting an application of real options theory with an attempt to capture the main properties of the underlying asset prices (Brennan and Schwartz [1985], Schwartz [1998]). In the case of an electricity generation asset, this is very rarely addressed as an optimal stopping time problem in dimension three (electricity, fuel and carbon prices). Nevertheless, some work in this direction is being undertaken (Fleten et al. [2007]).

Second, more realistic risk representations should now be investigated. The methods based on the computation of the right discount factor as in McDonald and Siegel [1986] could be considered a first way to take risk into account and still preserve a tractable problem. But this is neither the way discount factors are used in practice nor the way economic theory deals with risk preferences. The alternative method that consists of using a utility function provides a more rigorous approach to assess the effect of risk on investment timing. It has been developed more recently by Henderson [2007] and Grasselli [2011]. But if this approach is successful in dealing rationally with risks, it is too theoretical to succeed in convincing an investment board to take its decision based on a parametric utility function. The real risk of irreversible investment is to be stuck with a flow of profits that does not cover the fixed costs. However, few research papers deal with the valuation of an investment with a bankruptcy threshold whereas it is a common modelling framework in quantitative corporate finance (Rochet and Villeneuve [2005]). Moreover, for the electricity system in itself, the risk takes the form of a reliability threshold. The system should be designed in such a way that the probability that demand exceeds available production capacity should not exceed a certain small probability.

These kinds of constraints lead to stochastic target problems for which PDEs' characterisations are now available (Bouchard et al. [2010b,a]).

Third, if the first two points are to be developed, then there is little chance to be able to find analytical solutions and more results will have to rely on numerical methods. Generally the proposed numerical schemes for variational inequalities begin with the hypothesis that the user has given boundary conditions in the form of relationships between the value function and its derivatives at some points (see Dixit and Pindyck [1994, Appendix to chap. 10]). The alternative would be to develop more efficient forms of Howard's algorithm (Howard [1960]) as in Bokanowski et al. [2009].

Fourth, one cannot expect to convert financial division managers into masters of optimal control theory. One important economic research objective would be to find reduced investment rules that would mimic optimal investment rules. According to McDonald [1998], this is what corporate finance divisions using profitability index and payback time already do. But this is in an intuitive way that does not provide any knowledge of the *ex ante* error committed by using tese approximative rules. More could be done along this line.

Fifth and last, all this research would be of no interest if the sophisticated methods presented here were to provide only a negligible benefit. In a sense, the only thing that matters is if a firm's decision rule outperforms that of competitors'. Thus more *a posteriori* performance analysis as done by Driouchi and Bennett [2011] could be developed.

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