Speculative Trading of Electricity Contracts in Interconnected Locations

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joint work with

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For those interested in Algorithmic trading... available from Cambridge University Press (and Amazon)...

ALGORITHMIC AND HIGH-FREQUENCY TRADING

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Motivation

- Optimal investment problem:
  - How can an investor maximise expected profits by taking simultaneous positions in interconnected markets/locations?
  - How are positions dynamically adjusted over the trading horizon?
- Model of price dynamics
  - Investor must specify a model for electricity contracts.
  - How does the interconnector affect prices of electricity contracts?
- Model uncertainty:
  - How confident is the investor about the model?
  - How does model ambiguity/uncertainty affect trading strategies?
Outline

- Reference model for prices of electricity contract
- Price impact of trading: permanent and temporary
- Ambiguity aversion / Model uncertainty
- Class of candidate models
- Optimal trading strategy to trade in interconnected markets/locations
- Conclusions
Reference Model
Reference Model

In the **absence of interconnector**, the midprices \( P_t^i \) of (short-term) electricity contracts satisfy:

\[
dP_t^i = \kappa^i (\theta_t^i - P_t^i) \, dt + \sigma^i \, dW_t^i + dJ_t^i,
\]

where

- \( i \in \{1, 2\} \): location,
- \( \kappa^i, \sigma^i \geq 0 \),
- \( J_t^i \): compensated jump process,
- \( W_t^i \): standard Brownian motion, with correlation \( \rho \),
- \( \theta_t^i \): deterministic seasonal function.
Price Impact: permanent and temporary
Speed of trading and inventory

- **Speed of trading:**
  - $\nu_t$ in location 1,
  - $-\nu_t$ in location 2 (i.e. offsetting positions)

- **Over small time step $dt$ investor**
  - $\nu_t > 0$: **buys** $\nu_t dt$ in **location 1** and **sells** same amount in **location 2**
  - $\nu_t < 0$: **sells** $\nu_t dt$ in **location 1** and **buys** same amount in **location 2**

- $Q^\nu_t$ denotes inventory in **location 1**:

$$dQ^\nu_t = \nu_t dt, \quad Q^\nu_0 = 0.$$
Permanent Price Impact

- Investor’s trading activity affects prices of electricity contracts in both locations
- Market participants incorporate information of investor’s trading
  - Equilibrium prices will reflect the investor’s buying and selling pressure
- Permanent impact in the presence of interconnector

\[ dP_t^i = \kappa_i (\theta_t^i - P_t^i) dt + b_i \nu_t dt + \sigma_i dW_t^i + dJ_t^i, \]

where \( b_1 \geq 0 \) and \( b_2 \leq 0 \).

- **Permanent impact introduces mean-reversion** in prices
  - Investor tends to sell when prices are high
  - Investor tends to buy when prices are low
Temporary Price Impact

- Investor receives worse prices than those quoted at the time she trades in both locations
- **Temporary impact**

\[
\hat{P}_{t}^{1,\nu} = P_{t}^{1,\nu} + a_{1} \nu_{t},
\]

\[
\hat{P}_{t}^{2,\nu} = P_{t}^{2,\nu} - a_{2} \nu_{t},
\]

where \( a_{i} \geq 0 \).

- Elasticity of the liquidity of contracts supplied in both locations determines magnitude of \( a_{i} \)
Model Uncertainty

All models are wrong, but some are useful...
The diffusive class of measure changes is provided by the Radon-Nikodym derivative

$$\frac{dQ^\eta}{dP} = \exp \left\{ -\frac{1}{2} \int_0^T \eta'_u \Sigma^{-1} \eta_u \, du - \int_0^T \eta'_u \, dW_u \right\},$$

where $W_t = (W^X_t, W^\nu_t)'$, $\eta_t$ are $\mathcal{F}_t$-adapted processes, and

$$\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix},$$

so that

$$W^\eta_t = -\int_0^t \eta_u \, du + W_t,$$

are $Q^\eta$-standard Brownian motions.
Class of Candidate Models: Jumps

- The **jump** class of measure changes is provided by the Radon-Nikodym derivative

\[
\frac{dQ^g}{dP} = \exp \left\{ - \int_0^T \int_{-\infty}^{\infty} \left( e^{g_t(z)} - 1 \right) \nu_P(dz, dt) \right. \\
+ \int_0^T \int_{-\infty}^{\infty} g_t(z) \mu(dz, dt) \right\}
\]

- \(g_t\) is an \(\mathcal{F}_t\)-predictable random field so that **intensity and jump size** are **altered**

- The \(Q^g\) **mean-measure** of the PRM \(\mu(dz, dt)\) is

\[
\nu_{Q^g}(dz, dt) = e^{g_t(z)} \nu_P(dz, dt)
\]
Class of Candidate Models: Diffusions + Jumps

\[
\Phi, \eta_t \quad \xrightarrow{\mathcal{Q}_\eta} \quad \varepsilon, g_t(z) \quad \xleftarrow{\mathcal{Q}_g} \quad \Phi, \eta_t
\]

\[
\mathcal{P} \quad \xrightarrow{\varepsilon, g_t(z)} \quad \mathcal{Q}_g \quad \xleftarrow{\Phi, \eta_t}
\]

\(\Phi\) is an **ambiguity matrix**: 
\[
\Phi^{-1} = \frac{1}{\phi} \sum_{-1} + \frac{1}{\phi_1} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{\phi_2} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}
\]

The penalty is related to **relative entropy**:

\[
\mathbb{E}^{\mathcal{Q}_\eta,g} \left[ \mathcal{H}^{\Phi,\varepsilon} (\mathcal{Q}_\eta,g | \mathcal{P}) \right] = \mathbb{E}^{\mathcal{Q}_\eta,g} \left[ \frac{1}{2} \int_0^T \eta_t' \Phi^{-1} \eta_t \, dt \right]
\]

**diffusion ambiguity**

\[
+ \frac{1}{\varepsilon} \int_0^T \int_{-\infty}^{\infty} \left\{ 1 + e^{g_t(z)} (g_t(z) - 1) \right\} \nu_{\mathcal{P}}(dz, dt)
\]

**jump ambiguity**
Dynamic trading strategy
The investor seeks the optimal speed of trading $\nu$:

$$H = \sup_\nu \inf_{Q \in \mathcal{Q}} \mathbb{E}_t^Q \left[ \int_t^T f(s, P^1_s, P^2_s, \nu_s) \nu_s \, ds + g(T, P^1_T, P^2_T, Q_T) + H^{\Phi, \varepsilon}(Q|\mathcal{P}) \right].$$

- **Instantaneous profit**
  $$f(t, P^1_t, P^2_t, \nu_t) = ((P^2_t - a_2) - (P^1_t + a_1)) \nu_t.$$

- **Liquidation of contracts at time $T$**
  $$g(T, P^1_T, P^2_T, q_T) = (P^1_T - P^2_T) q_T - \alpha q_T^2, \quad \alpha \gg 0.$$
Proposition. The dynamic pricing equation associated to the value function reduces to

\[
(\partial_t + \mathcal{L})H + \frac{1}{4A} \left[ (b_1 \partial_{p_1} - b_2 \partial_{p_2} + \partial_q)H + (P^2 - P^1) \right]^2 \\
- \frac{1}{2} \mathcal{D}H' \Omega' \Phi^{-1} \Omega \mathcal{D}H + \sum_{i=1}^{2} \lambda_i \int_{-\infty}^{\infty} \frac{1 - e^{-\varepsilon \Delta_i(y)H}}{\varepsilon} G_i(dy) = 0 ,
\]

subject to the terminal condition

\[
H(T, P^1, P^2, q) = (P^1_T - P^2_T) q - \alpha q^2 ,
\]

where

\[
\Omega = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}, \quad \mathcal{D}H = \begin{pmatrix} \partial_{p_1} H \\ \partial_{p_2} H \end{pmatrix} ,
\]

and

\[
\Delta_i(y)H = H(t, P^i + y, q) - H(t, P^i, q) .
\]
Optimal trading strategy

- The optimal controls in feedback form are:

\[ \nu^*(t, P^1, P^2, q) = \frac{1}{2a} (\xi_0(t) + \xi_1(t) P^1 + \xi_2(t) P^2 + \xi_3(t) q) , \]

where \( a = a_1 + a_2 \).

- The optimal diffusion and jump adjustments are

\[ \eta^*(t, P^1, P^2, q) = -\Phi \Omega \mathcal{D} H , \]
\[ g_i^*(t, P^1, P^2, y) = -\epsilon \Delta_i(y) H . \]
Ambiguity to Diffusion Only
**Ambiguity to diffusion only**

**PIDE for Interconnector Trading Under Diffusion Ambiguity.**

Let $\epsilon \downarrow 0$ and $\phi > 0$, so the investor is ambiguity averse only to the diffusive component of the reference measure $\mathbb{P}$. Then the PIDE (1) reduces to

$$
(\partial_t + \mathcal{L})H + \frac{1}{4a} \left[ (b_1 \partial P_1 - b_2 \partial P_2 + \partial q)H + (P^2 - P^1) \right]^2
$$

$$
- \frac{1}{2} \mathcal{D} H' \Omega^{-1} \Omega \mathcal{D} H + \sum_{i=1}^{2} \lambda_i \int_{-\infty}^{\infty} \Delta H_i G_i(dy) = 0.
$$

The optimal trading strategy is

$$
\nu^* = \frac{1}{2a} \left( I_0^*(t) + I_1^*(t) P^1 + I_2^*(t) P^2 + I_3^*(t) q \right).
$$
Under the optimal measure, the investor ‘assumes’ that prices satisfy:

\[
\begin{align*}
    dP_1^t &= \kappa_1 \left( \theta_1^t - \frac{\phi}{\kappa_1} \sigma_1 (\partial P_1^1 H + \rho \partial P_2^2 H) - P_1^t \right) dt + b_1 \nu_t dt + \sigma_1 dW_{1*}^t, \\
    dP_2^t &= \kappa_2 \left( \theta_2^t - \frac{\phi}{\kappa_2} \sigma_2 (\rho \partial P_1^1 H + \partial P_2^2 H) - P_2^t \right) dt - b_2 \nu_t dt + \sigma_2 dW_{2*}^t.
\end{align*}
\]

where

\[
\begin{align*}
    \partial P_1^1 H &= l_{01}^* + 2l_{021}^* P_1^1 + l_{012}^* P_2^1 + (l_{11}^* + 1)q, \\
    \partial P_2^2 H &= l_{02}^* + 2l_{022}^* P_2^2 + l_{012}^* P_1^2 + (l_{12}^* - 1)q.
\end{align*}
\]
Drift adjustment

Figure: Optimal drift $\eta^*_1 = -\phi \sigma_1 (\partial_{P_1} H + \rho \partial_{P_2} H)$, $\eta^*_2 = -\phi \sigma_2 (\rho \partial_{P_1} + \partial_{P_2}) H$, $P^1 = 19$, $P^2 = 21$, $Q = 0$, $\phi = 10^{-6}$. 
The HJBI (1) is nonlinear and we cannot obtain a solution in closed-form, so we employ perturbation methods to approximate the value function employing the expansion

$$H(t, P, q) = H_0(t, P, q) + \phi H_D(t, P, q) + \varepsilon H_J(t, P, q) + O(\nu),$$

where $\nu = \max(\phi^2, \phi \varepsilon, \varepsilon^2)$. 
Solving for $H_0(t, P, q)$

**Proposition** In the limit $(\varepsilon, \phi) \downarrow (0, 0)$, the value function of the ambiguity neutral investor, i.e. $H_0(t, P, q)$, satisfies the PIDE

$$0 = (\partial_t + \mathcal{L}) H_0 + \frac{1}{4a} \left[ (b_1 \partial P_1 - b_2 \partial P_2 + \partial_q) H_0 + (P^2 - P^1) \right]^2$$

$$+ \sum_{i=1,2} \lambda_i \int_{-\infty}^{\infty} \Delta_i(y) H_0 \, G_i(dy),$$

subject to $H_0(T, P^1, P^2, q) = (P^1 - P^2) \, q - \alpha \, q^2$, and admits the ansatz

$$H_0(t, P, q) = \ell_0^{(0)}(t) + \ell_0^{(0)\top}(t) \, P + P^\top \ell_{01}^{(0)} \, P$$

$$+ \left( \ell_{10}^{(0)}(t) + (P^1 - P^2) + \ell_1^{(0)\top}(t) \, P \right) \, q + \ell_2(t) \, q^2,$$

where $\ell_0^{(0)}(t)$, $\ell_{01}^{(0)}(t)$, $\ell_1^{(0)}(t)$ are vector and matrix-valued deterministic functions of time, which satisfy the ODE system.
Solving for $H_D(t, P, q)$

**Proposition** $H_D(t, P, q)$ satisfies

$$(\partial_t + \mathcal{L}) H_D + f_D(t, P, q) \left( b_1 \partial_{P_1} - b_2 \partial_{P_2} + \partial_q \right) H_D + \sum_{i=1}^{2} \lambda_i \int_{-\infty}^{\infty} \Delta_i(y) H_D G_i(dy)$$

$$- \frac{1}{2\phi} \mathcal{D} H_0' \Omega' \Phi^{-1} \Omega \mathcal{D} H_0 = 0,$$

with terminal condition $H_D(T, P^1, P^2, q) = 0$, where

$$f_D(t, P, q) = \frac{1}{2a} \left[ (b_1 \partial_{P_1} - b_2 \partial_{P_2} + \partial_q) H_0(t, P, q) + (P^2 - P^1) \right].$$

Equation admits the ansatz

$$H_D(t, P, q) = \ell_0^{(0)}(t) + \ell_0^{(0)\top}(t) P + P^\top \ell_{01}^{(0)} P + \left( \ell_{10}^{(0)}(t) + \ell_1^{(0)\top}(t) P \right) q + \ell_2(t) q^2,$$

where $\ell_0^{(0)}(t)$, $\ell_{01}^{(0)}(t)$, $\ell_1^{(0)}(t)$ are vector and matrix-valued deterministic functions of time.
Solving for $H_J(t, P, q)$

**Proposition** $H_J(t, P, q)$ satisfies

$$(\partial_t + L)H_J + f_H(t, P, q) \left( b_1 \partial p_1 - b_2 \partial p_2 + \partial q \right) H_J$$

$$+ \sum_{i=1}^{2} \lambda_i \int_{-\infty}^{\infty} \left( \Delta_i(y)H_J - \frac{1}{2} (\Delta_i(y)H_0)^2 \right) G_i(dy) = 0,$$

subject to $H_J(T, P, q) = 0$, where

$$f_H(t, P, q) = \frac{1}{2a} \left[ (b_1 \partial p_1 - b_2 \partial p_2 + \partial q) H_0(t, P, q) + (P^2 - P^1) \right].$$

Equation above admits the ansatz

$$H_J(t, P, q) = \ell_0^{(0)}(t) + \ell_0^{(0)T}(t) P + P^T \ell_{01}^{(0)} P + \left( \ell_{10}^{(0)}(t) + \ell_{11}^{(0)}(t) P \right) q + \ell_2(t) q^2,$$

where $\ell_0^{(0)}(t)$, $\ell_{01}^{(0)}(t)$, $\ell_{11}^{(0)}(t)$ are vector and matrix-valued deterministic functions of time.
Performance of Strategy
Simulations

- Perform 10,000 simulations
- Midprices of contracts are simulated under the statistical measure $\tilde{\mathbb{P}}$
- Jumps in both locations is the double exponential distribution:

$$G_i(dy) = \left\{ p_i m_i^+ e^{-m_i^+ y} \mathbb{1}_{y>0} + (1-p_i) m_i^- e^{-m_i^- |y|} \mathbb{1}_{y\leq0} \right\} \, dy,$$

where $m_i^\pm > 0$, $\lambda_i$ are the arrival rate of jumps, and $p_i$ are the probabilities of upwards jumps in midprices in each location.

| Model | $\kappa_1$ | $\kappa_2$ | $\sigma_1$ | $\sigma_2$ | $\rho$ | $\theta_1$ | $\theta_2$ | $m_1^+$ | $m_2^+$ | $m_1^-$ | $m_2^-$ | $\lambda_1$ | $\lambda_2$ | $p_1$ | $p_2$
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Prices and optimal trading in location 1

Figure: Sample Trading path and speed of trading under diffusive ambiguity
**Performance of strategy**

**Figure** : Sharpe ratios and percentiles of inventory holdings in location 1 for different values of $\phi$ and $\varepsilon \downarrow 0$
Performance of strategy

Figure: Sharpe Ratio for different values $\varepsilon > 0$, $\phi > 0$
Summary and ongoing

- Provided a model to trade in interconnected markets
- Accounted for price impact:
  - Short-term effect of supply stack and demand
  - Permanent

Ongoing:
- Calibrate to market data: different contracts, locations, and contract maturities.
- Incorporating model ambiguity on tree models and swing option valuation
Thanks for your attention!!!