Speculative Trading of Electricity Contracts in Interconnected Locations

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joint work with

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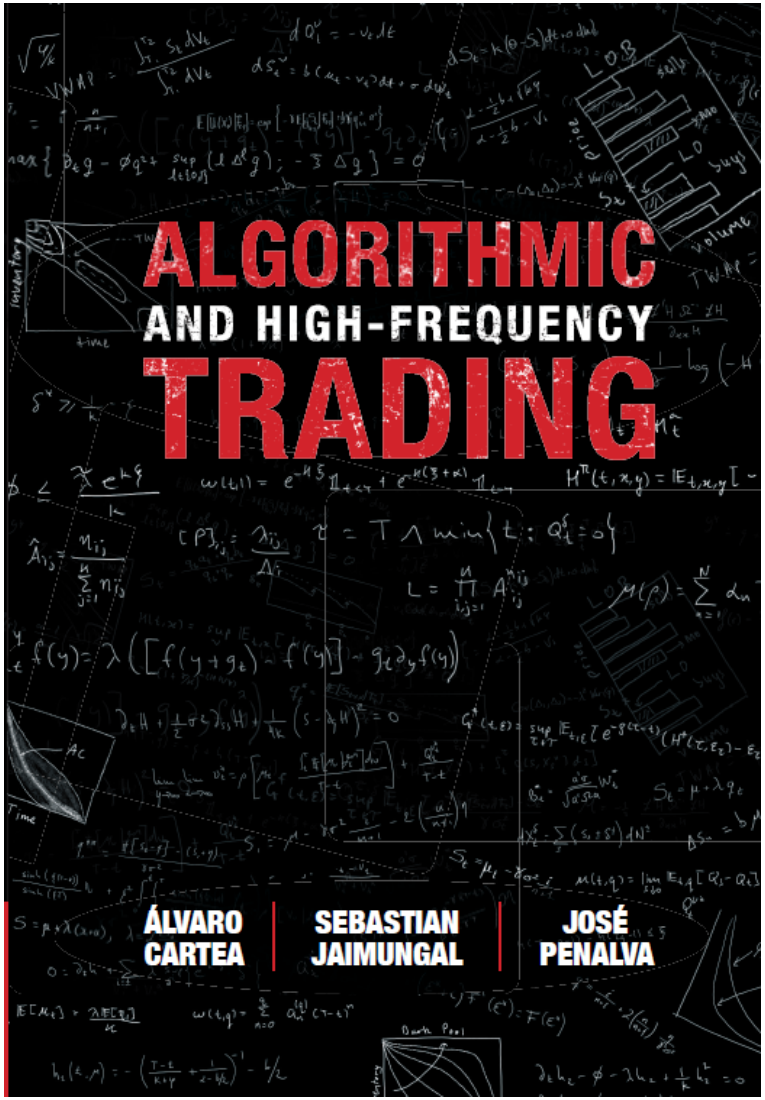
For those interested in Algorithmic trading... available from Cambridge University Press (and Amazon)...



Motivation

- Optimal investment problem:
  - How can an investor maximise expected profits by taking simultaneous positions in interconnected markets/locations?
  - How are positions dynamically adjusted over the trading horizon?
- Model of price dynamics
  - Investor must specify a model for electricity contracts.
  - How does the interconnector affect prices of electricity contracts?
- Model uncertainty:
  - How confident is the investor about the model?
  - How does model ambiguity/uncertainty affect trading strategies?

Outline

- Reference model for prices of electricity contract
- Price impact of trading: permanent and temporary
- Ambiguity aversion / Model uncertainty
- Class of candidate models
- Optimal trading strategy to trade in interconnected markets/locations
- Conclusions

Reference Model

Reference Model

In the **absence of interconnector**, the midprices P_t^i of (short-term) electricity contracts satisfy:

$$dP_t^i = \kappa^i(\theta_t^i - P_t^i)\,dt + \sigma^i\,dW_t^i + \,dJ_t^i\,,$$

where

- $i \in \{1, 2\}$: location,
- κ^i, $\sigma^i \geq 0$,
- J_t^i: compensated jump process,
- W_t^i: standard Brownian motion, with correlation ρ,
- θ_t^i: deterministic seasonal function.

Price Impact: permanent and temporary

Speed of trading and inventory

- Speed of trading:
  - ν_t in location 1,
  - $-\nu_t$ in location 2 (i.e. offsetting positions)
- Over small time step dt investor
  - $\nu_t > 0$: **buys** $\nu_t\, dt$ in **location 1** and **sells** same amount in **location 2**
  - $\nu_t < 0$: **sells** $\nu_t\, dt$ in **location 1** and **buys** same amount in **location 2**
- Q_t^ν denotes inventory in **location 1**:

$$dQ_t^\nu = \nu_t\, dt\,, \qquad Q_0^\nu = 0\,.$$

Permanent Price Impact

- Investor's trading activity affects prices of electricity contracts in both locations
- Market participants incorporate information of investor's trading
  - Equilibrium prices will reflect the investor's buying and selling pressure
- Permanent impact in the **presence of interconnector**

$$dP_t^i = \kappa_i \left(\theta_t^i - P_t^i\right) dt + \mathbf{b_i}\, \nu_t\, dt + \sigma_i\, dW_t^i + \, dJ_t^i \,,$$

  where $\mathbf{b_1} \geq 0$ and $\mathbf{b_2} \leq 0$.
- **Permanent impact introduces mean-reversion** in prices
  - Investor tends to sell when prices are high
  - Investor tends to buy when prices are low

Temporary Price Impact

- Investor receives worse prices than those quoted at the time she trades in both locations
- **Temporary impact**

$$\hat{P}_t^{1,\nu} = P_t^{1,\nu} + \mathbf{a_1}\,\nu_t\,,$$
$$\hat{P}_t^{2,\nu} = P_t^{2,\nu} - \mathbf{a_2}\,\nu_t\,,$$

  where $\mathbf{a_i} \geq 0$.
- Elasticity of the liquidity of contracts supplied in both locations determines magnitude of $\mathbf{a_i}$

Model Uncertainty

All models are wrong, but some are useful...

Class of Candidate Models: Diffusion

- The **diffusive** class of measure changes is provided by the Radon-Nikodym derivative

$$\frac{d\mathbb{Q}^{\boldsymbol{\eta}}}{d\mathbb{P}} = \exp\left\{ -\frac{1}{2}\int_0^T \boldsymbol{\eta}_u' \, \Sigma^{-1} \, \boldsymbol{\eta}_u \, du - \int_0^T \boldsymbol{\eta}_u' \, d\boldsymbol{W}_u \right\} ,$$

where $\boldsymbol{W}_t = (W_t^X, \; W_t^v)'$, $\boldsymbol{\eta}_t$ are $\mathcal{F}_t$-adapted processes, and

$$\boldsymbol{\Sigma} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} ,$$

so that

$$\boldsymbol{W}_t^{\boldsymbol{\eta}} = -\int_0^t \boldsymbol{\eta}_u \, du + \boldsymbol{W}_t ,$$

are $\mathbb{Q}^{\boldsymbol{\eta}}$-standard Brownian motions.

Class of Candidate Models: Jumps

- The **jump** class of measure changes is provided by the Radon-Nikodym derivative

$$\frac{d\mathbb{Q}^g}{d\mathbb{P}} = \exp\left\{ -\int_0^T \int_{-\infty}^{\infty} \left(e^{\mathbf{g_t(z)}} - 1 \right) \nu_{\mathbb{P}}(dz, dt) + \int_0^T \int_{-\infty}^{\infty} \mathbf{g_t(z)} \, \mu(dz, dt) \right\}$$

  - $\mathbf{g_t}$ is an $\mathcal{F}_t$-predictable random field so that **intensity and jump size** are **altered**
  - The $\mathbb{Q}^g$ **mean-measure** of the PRM $\mu(dz, dt)$ is

$$\boxed{\nu_{\mathbb{Q}^g}(dz, dt) = e^{\mathbf{g_t(z)}} \, \nu_{\mathbb{P}}(dz, dt)}$$

Class of Candidate Models: Diffusions+Jumps

$$
\begin{tikzcd}
 & \mathbb{Q}^{\boldsymbol{\eta}} \arrow[dr, "\varepsilon, g_t(z)"] & \\
\mathbb{P} \arrow[ur, "\boldsymbol{\Phi}, \boldsymbol{\eta}_t"] \arrow[rr, dotted] \arrow[dr, "\varepsilon, g_t(z)"'] & & \mathbb{Q}^{\boldsymbol{\eta},g} \\
 & \mathbb{Q}^{g} \arrow[ur, "\boldsymbol{\Phi}, \boldsymbol{\eta}_t"'] &
\end{tikzcd}
$$

$\boldsymbol{\Phi}$ is an **ambiguity matrix**: $\boldsymbol{\Phi}^{-1} = \frac{1}{\phi}\Sigma^{-1} + \frac{1}{\phi_1}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{\phi_2}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

The penalty is related to **relative entropy**:

$$
\mathbb{E}^{\mathbb{Q}^{\boldsymbol{\eta},\boldsymbol{g}}}\left[\mathcal{H}^{\boldsymbol{\Phi},\varepsilon}(\mathbb{Q}^{\boldsymbol{\eta},\boldsymbol{g}}|\mathbb{P})\right] = \mathbb{E}^{\mathbb{Q}^{\boldsymbol{\eta},\boldsymbol{g}}}\Bigg[\overbrace{\frac{1}{2}\int_0^T \boldsymbol{\eta}_t' \boldsymbol{\Phi}^{-1} \boldsymbol{\eta}_t \, dt}^{\textbf{diffusion ambiguity}}
$$

$$
\underbrace{+\frac{1}{\varepsilon}\int_0^T \int_{-\infty}^{\infty} \left\{1 + e^{g_t(z)}\left(g_t(z) - 1\right)\right\} \nu_{\mathbb{P}}(dz, dt)}_{\textbf{jump ambiguity}}\Bigg]
$$

Dynamic trading strategy

Value function

The investor seeks the optimal speed of trading ν:

$$H = \sup_{\nu} \inf_{\mathbb{Q}\in\mathcal{Q}} \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^T \underbrace{f(s, P_s^1, P_s^2, \nu_s)}_{\text{cashflow}} \nu_s \, ds + \underbrace{g(T, P_T^1, P_T^2, Q_T)}_{\text{terminal liquidation}} + \underbrace{\mathcal{H}^{\mathbf{\Phi},\varepsilon}(\mathbb{Q}|\mathbb{P})}_{\text{model penalty}} \right] .$$

- Instantaneous profit

$$f(t, P_t^1, P_t^2, \nu_t) = \left((P_t^2 - \mathbf{a_2}) - (P_t^1 + \mathbf{a_1}) \right) \nu_t \, .$$

- Liquidation of contracts at time T

$$g(T, P_T^1, P_T^2, q_T) = (P_T^1 - P_T^2)\, q_T - \alpha\, q_T^2 \, , \qquad \alpha \gg 0 \, .$$

DPE

Proposition. The **dynamic pricing equation** associated to the value function reduces to

$$(\partial_t + \mathcal{L})H + \frac{1}{4\,A}\left[(b_1\partial_{P^1} - b_2\partial_{P^2} + \partial_q)H + (P^2 - P^1)\right]^2 - \frac{1}{2}\,\boldsymbol{\mathcal{D}}H'\,\boldsymbol{\Omega}'\,\boldsymbol{\Phi}^{-1}\,\boldsymbol{\Omega}\,\boldsymbol{\mathcal{D}}H + \sum_{i=1}^{2}\lambda_i\int_{-\infty}^{\infty}\frac{1-e^{-\varepsilon\Delta_i(y)H}}{\varepsilon}\,G_i(dy) = 0\,, \tag{1}$$

subject to the terminal condition

$$H(T, P^1, P^2, q) = (P_T^1 - P_T^2)\,q - \alpha\,q^2\,,$$

where

$$\boldsymbol{\Omega} = \begin{pmatrix}\sigma_1 & 0\\ 0 & \sigma_2\end{pmatrix}, \quad \boldsymbol{\mathcal{D}}H = \begin{pmatrix}\partial_{P^1}H\\ \partial_{P^2}H\end{pmatrix}\,,$$

and

$$\Delta_i(y)H = H(t, P^i + y, q) - H(t, P^i, q)\,.$$

Optimal trading strategy

- The optimal controls in feedback form are:

$$\nu^*(t, P^1, P^2, q) = \frac{1}{2\,a}\left(\xi_0(t) + \xi_1(t)\,\boldsymbol{P^1} + \xi_2(t)\,\boldsymbol{P^2} + \xi_3(t)\,\boldsymbol{q}\right)\,,$$

where $a = a_1 + a_2$.

- The optimal diffusion and jump adjustments are

$$\begin{aligned} \boldsymbol{\eta}^*(t, P^1, P^2, q) &= -\boldsymbol{\Phi}\,\boldsymbol{\Omega}\,\boldsymbol{\mathcal{D}}\,H\,, \\ g_i^*(t, P^1, P^2, y) &= -\varepsilon\,\Delta_i(y)H\,. \end{aligned}$$

Ambiguity to Diffusion Only

Ambiguity to diffusion only

PIDE for Interconnector Trading Under Diffusion Ambiguity. Let $\varepsilon \downarrow 0$ and $\phi > 0$, so the investor is ambiguity averse only to the diffusive component of the reference measure $\mathbb{P}$. Then the PIDE (1) reduces to

$$(\partial_t + \mathcal{L})H + \frac{1}{4\,a}\left[(b_1\partial_{P^1} - b_2\partial_{P^2} + \partial_q)H + (P^2 - P^1)\right]^2 - \frac{1}{2}\,\mathcal{D}H'\,\mathbf{\Omega}'\,\mathbf{\Phi}^{-1}\,\mathbf{\Omega}\,\mathcal{D}H + \sum_{i=1}^{2}\lambda_i \int_{-\infty}^{\infty} \Delta H_i\, G_i(dy) \;=\; 0\,.$$

The optimal trading strategy is

$$\nu^* = \frac{1}{2\,a}\left(\mathfrak{l}_0^*(t) + \mathfrak{l}_1^*(t)\,P^1 + \mathfrak{l}_2^*(t)\,P^2 + \mathfrak{l}_3^*(t)\,q\right)\,.$$

Price dynamics under optimal measure

Under the optimal measure, the investor 'assumes' that prices satisfy:

$$
\begin{aligned}
dP_t^1 &= \kappa_1 \left(\theta_t^1 - \frac{\phi}{\kappa_1} \sigma_1 \left(\partial_{P^1} H + \rho \, \partial_{P^2} H \right) - P_t^1 \right) dt + b_1 \, \nu_t \, dt + \sigma_1 \, dW_t^{1*} \, , \\
dP_t^2 &= \kappa_2 \left(\theta_t^2 - \frac{\phi}{\kappa_2} \sigma_2 \left(\rho \, \partial_{P^1} H + \partial_{P^2} H \right) - P_t^2 \right) dt - b_2 \, \nu_t \, dt + \sigma_2 \, dW_t^{2*} \, .
\end{aligned}
$$

where

$$
\begin{aligned}
\partial_{P^1} H &= l_{01}^* + 2 l_{021}^* P^1 + l_{012}^* P^2 + (l_{11}^* + 1) q \, , \\
\partial_{P^2} H &= l_{02}^* + 2 l_{022}^* P^2 + l_{012}^* P^1 + (l_{12}^* - 1) q \, .
\end{aligned}
$$

Drift adjustment

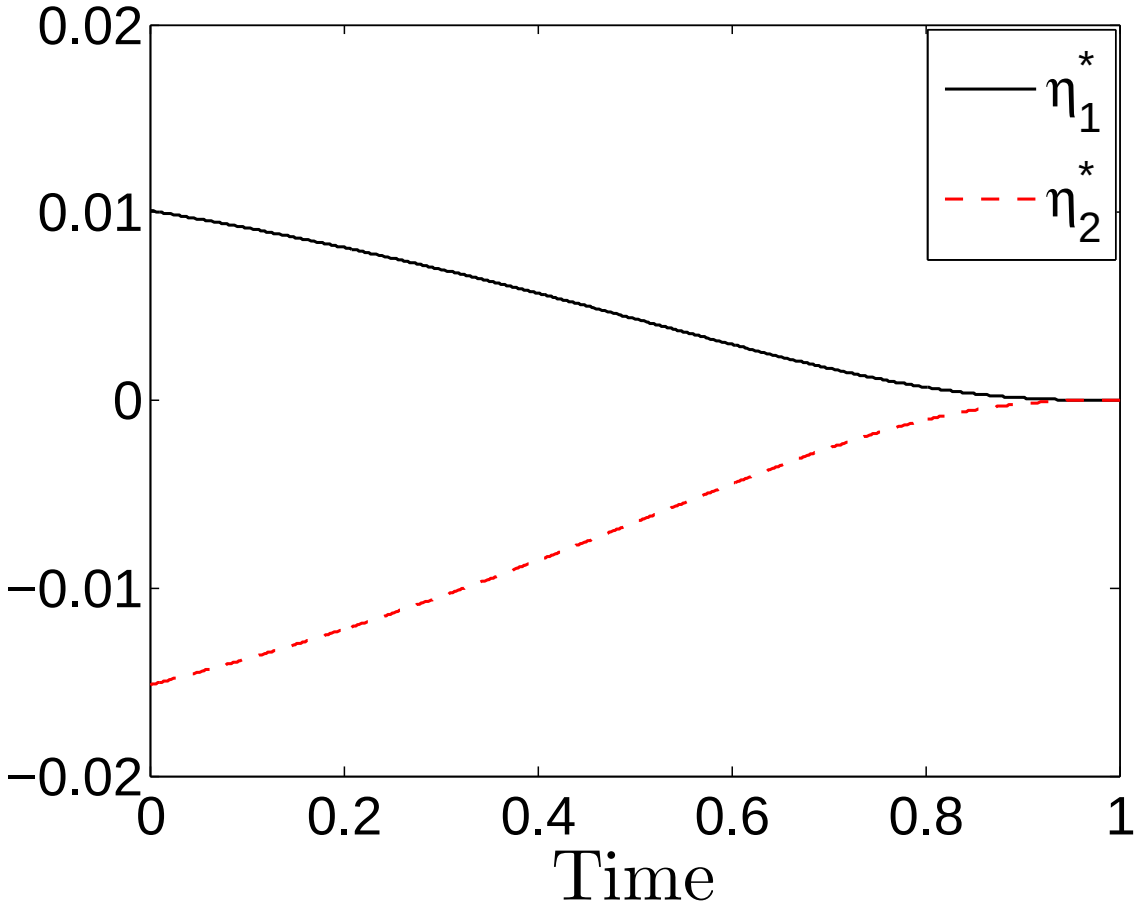


Figure : Optimal drift $\eta_t^{1*} = -\phi\,\sigma_1\,(\partial_{P_1} H + \rho\,\partial_{P_2} H)$, $\eta_t^{2*} = -\phi\,\sigma_2\,(\rho\,\partial_{P_1} + \partial_{P_2})\,H$, $P^1 = 19$, $P^2 = 21$, $Q = 0$, $\phi = 10^{-6}$.

Asymptotic expansion

The HJBI (1) is nonlinear and we cannot obtain a solution in closed-form, so we employ perturbation methods to approximate the value function employing the expansion

$$H(t, \boldsymbol{P}, q) = H_0(t, \boldsymbol{P}, q) + \phi\, H_D(t, \boldsymbol{P}, q) + \varepsilon\, H_J(t, \boldsymbol{P}, q) + O(\upsilon)\,,$$

where $\upsilon = \max(\phi^2,\, \phi\,\varepsilon,\, \varepsilon^2)$.

Solving for $H_0(t, \boldsymbol{P}, q)$

Proposition In the limit $(\varepsilon, \phi) \downarrow (0,0)$, the value function of the ambiguity neutral investor, i.e. $H_0(t, \boldsymbol{P}, q)$, satisfies the PIDE

$$0 = (\partial_t + \mathcal{L})\, H_0 + \tfrac{1}{4\,a}\, \left[(b_1\, \partial_{P^1} - b_2\, \partial_{P^2} + \partial_q)\, H_0 + \left(P^2 - P^1\right)\right]^2 + \sum_{i=1,2} \lambda_i \int_{-\infty}^{\infty} \Delta_i(y) H_0\, G_i(dy)\,,$$

subject to $H_0(T, P^1, P^2, q) = \left(P^1 - P^2\right)\, q - \alpha\, q^2$, and admits the ansatz

$$H_0(t, \boldsymbol{P}, q) = \ell_0^{(0)}(t) + \boldsymbol{\ell}_0^{(0)\intercal}(t)\, \boldsymbol{P} + \boldsymbol{P}^\intercal \boldsymbol{\ell}_{01}^{(0)} \boldsymbol{P} + \left(\ell_{10}^{(0)}(t) + (P^1 - P^2) + \boldsymbol{\ell}_1^{(0)\intercal}(t) \boldsymbol{P}\right)\, q + \ell_2(t)\, q^2\,,$$

where $\boldsymbol{\ell}_0^{(0)}(t)$, $\boldsymbol{\ell}_{01}^{(0)}(t)$, $\boldsymbol{\ell}_1^{(0)}(t)$ are vector and matrix-valued deterministic functions of time, which satisfy the ODE system.

Solving for $H_D(t, \boldsymbol{P}, q)$

Proposition $H_D(t, \boldsymbol{P}, q)$ satisfies

$$(\partial_t + \mathcal{L})\, H_D + f_D(t, \boldsymbol{P}, q)\, (b_1\, \partial_{P^1} - b_2\, \partial_{P^2} + \partial_q)\, H_D + \sum_{i=1}^{2} \lambda_i \int_{-\infty}^{\infty} \Delta_i(y) H_D\, G_i(dy)$$
$$- \frac{1}{2\phi}\, \boldsymbol{\mathcal{D}} H_0'\, \boldsymbol{\Omega}'\, \boldsymbol{\Phi}^{-1}\, \boldsymbol{\Omega}\, \boldsymbol{\mathcal{D}} H_0 = 0\,,$$

with terminal condition $H_D(T, P^1, P^2, q) = 0$, where

$$f_D(t, \boldsymbol{P}, q) = \frac{1}{2\,a} \left[(b_1\, \partial_{P^1} - b_2\, \partial_{P^2} + \partial_q)\, H_0(t, \boldsymbol{P}, q) + \left(P^2 - P^1\right) \right]\,.$$

Equation admits the ansatz

$$H_D(t, \boldsymbol{P}, q) = \ell_0^{(0)}(t) + \boldsymbol{\ell}_0^{(0)\intercal}(t)\, \boldsymbol{P} + \boldsymbol{P}^\intercal \boldsymbol{\ell}_{01}^{(0)} \boldsymbol{P} + \left(\ell_{10}^{(0)}(t) + \boldsymbol{\ell}_1^{(0)\intercal}(t) \boldsymbol{P} \right)\, q + \ell_2(t)\, q^2\,,$$

where $\boldsymbol{\ell}_0^{(0)}(t)$, $\boldsymbol{\ell}_{01}^{(0)}(t)$, $\boldsymbol{\ell}_1^{(0)}(t)$ are vector and matrix-valued deterministic functions of time.

Solving for $H_J(t, \boldsymbol{P}, q)$

Proposition $H_J(t, \boldsymbol{P}, q)$ satisfies

$$(\partial_t + \mathcal{L})H_J + f_H(t, \boldsymbol{P}, q)\,\left(b_1\,\partial_{P^1} - b_2\,\partial_{P^2} + \partial_q\right)\,H_J$$
$$+ \sum_{i=1}^{2} \lambda_i \int_{-\infty}^{\infty} \left(\Delta_i(y)H_J - \frac{1}{2}(\Delta_i(y)H_0)^2\right)\,G_i(dy) = 0\,,$$

subject to $H_J(T, \boldsymbol{P}, q) = 0$, where

$$f_H(t, \boldsymbol{P}, q) = \frac{1}{2\,a}\left[\left(b_1\,\partial_{P^1} - b_2\,\partial_{P^2} + \partial_q\right)H_0(t, \boldsymbol{P}, q) + \left(P^2 - P^1\right)\right]\,.$$

Equation above admits the ansatz

$$H_J(t, \boldsymbol{P}, q) = \ell_0^{(0)}(t) + \boldsymbol{\ell}_0^{(0)\intercal}(t)\,\boldsymbol{P} + \boldsymbol{P}^\intercal \boldsymbol{\ell}_{01}^{(0)}\boldsymbol{P} + \left(\ell_{10}^{(0)}(t) + \boldsymbol{\ell}_1^{(0)\intercal}(t)\boldsymbol{P}\right)\,q + \ell_2(t)\,q^2\,,$$

where $\boldsymbol{\ell}_0^{(0)}(t)$, $\boldsymbol{\ell}_{01}^{(0)}(t)$, $\boldsymbol{\ell}_1^{(0)}(t)$ are vector and matrix-valued deterministic functions of time.

Performance of Strategy

Simulations

- Perform 10,000 simulations
- Midprices of contracts are simulated under the statistical measure $\widetilde{\mathbb{P}}$
- Jumps in both locations is the double exponential distribution:

$$G_i(dy) = \left\{ p_i \, m_i^+ \, e^{-m_i^+ y} \, \mathbb{1}_{z>0} + (1-p_i) \, m_i^- \, e^{-m_i^- |y|} \, \mathbb{1}_{y \le 0} \right\} dy \, ,$$

where $m_i^{\pm} > 0$, λ_i are the arrival rate of jumps, and p_i are the probabilities of upwards jumps in midprices in each location.

Model	κ_1	κ_2	σ_1	σ_2	ρ	θ_1	θ_2	m_1^+	m_2^+	m_1^-	m_2^-	λ_1	λ_2	p_1	p_2
$\mathbb{P}$	3.5	3.5	1	1.5	0.5	20	20	0.7	0.7	0.3	0.3	2	2	0.6	0.6
$\widetilde{\mathbb{P}}$	3.0	2.5	1	1.25	0.6	20	20	0.6	0.6	0.4	0.4	2.5	2.5	0.65	0.65

Prices and optimal trading in location 1

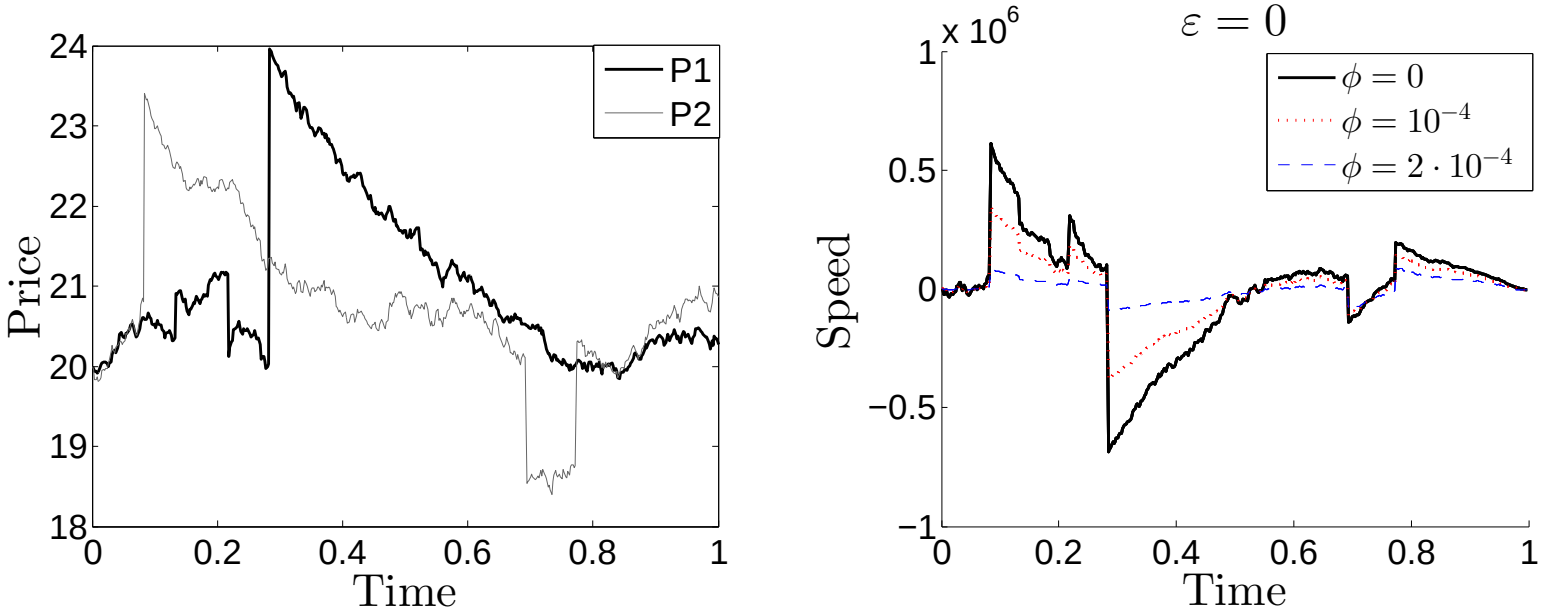
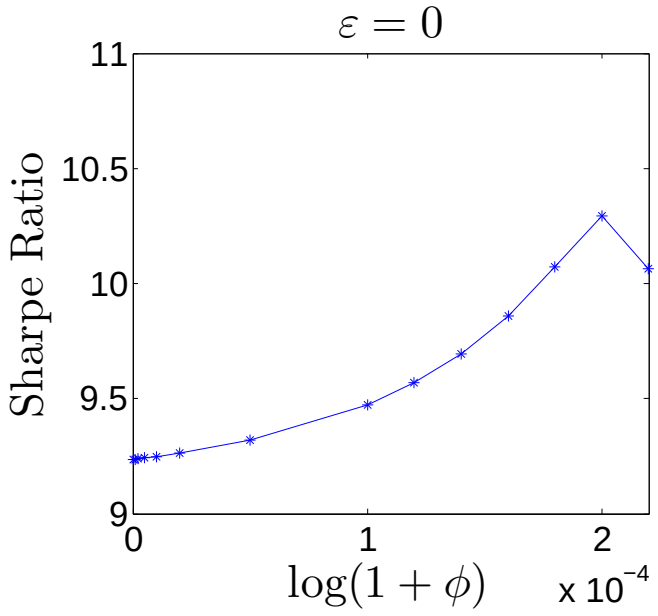


Figure : Sample Trading path and speed of trading under diffusive ambiguity

Performance of strategy



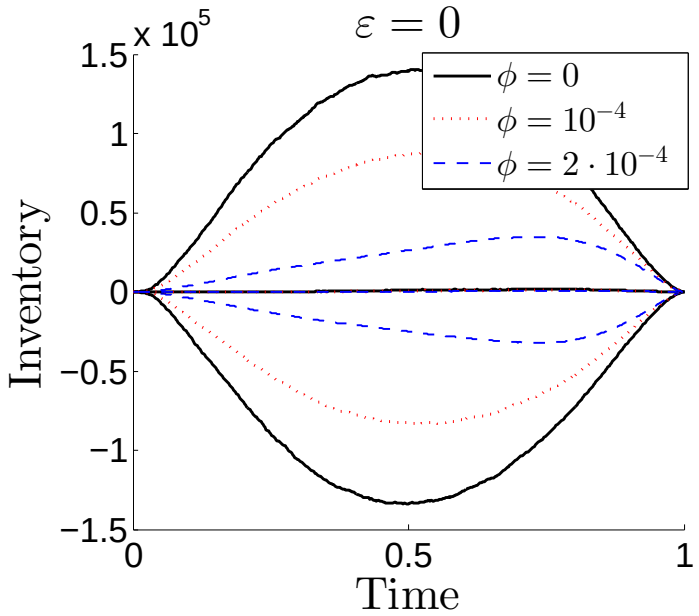


Figure : Sharpe ratios and percentiles of inventory holdings in location 1 for different values of ϕ and $\varepsilon \downarrow 0$

Performance of strategy

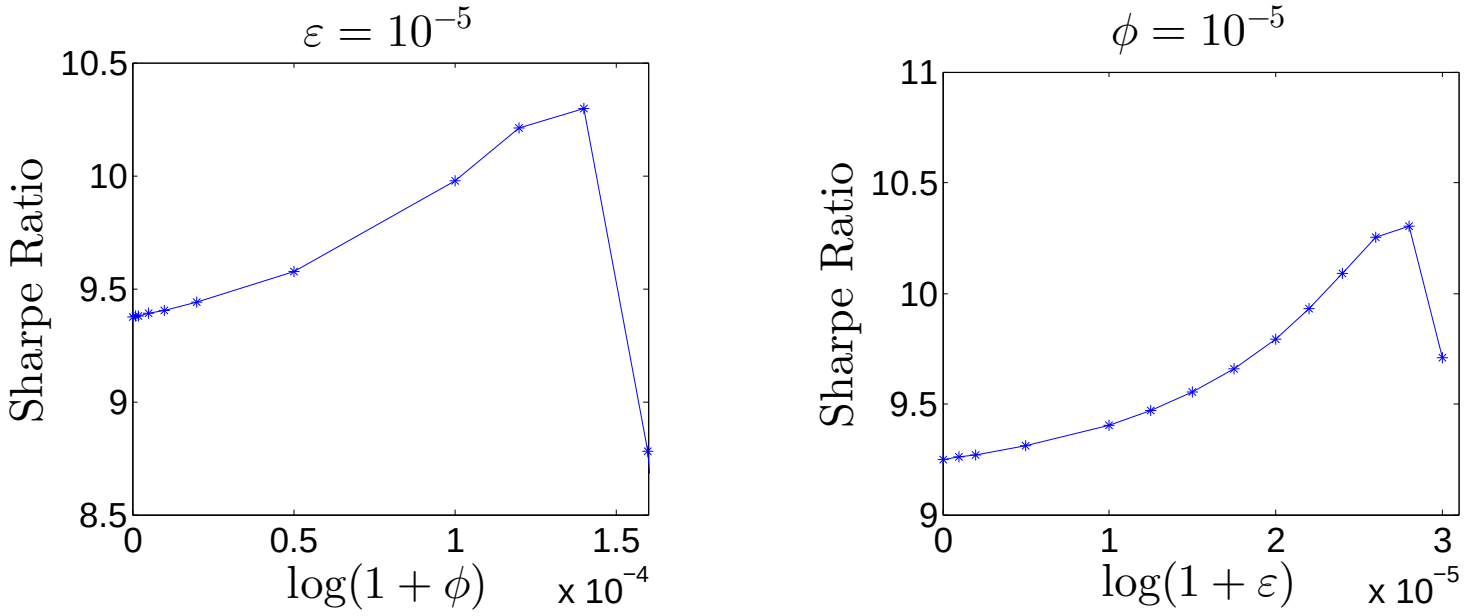


Figure : Sharpe Ratio for different values $\varepsilon > 0$, $\phi > 0$

Summary and ongoing

- Provided a model to trade in interconnected markets
- Accounted for price impact:
  - Short-term effect of supply stack and demand
  - Permanent

Ongoing:

- Calibrate to market data: different contracts, locations, and contract maturities.
- Incorporating model ambiguity on tree models and swing option valuation

Thanks for your attention!!!