Optimal Dynamic Regulation of Carbon Emissions Market

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Agenda

Motivations

Motivations

- A significant part of carbon emissions in the EU falls in the EU Trading Systems of carbon allowances launched in 2005.
- We are now in Phase IV (2013-2020).
- Recent introduction of Stability Market Reserve to prevent the market from being too long or too short through either backloading or auctions.
- If the total number of allowances under circulation falls under 400 milllions, the regulator adds allowances. If it reaches 800 million, allowances are withdrawn.
- Makes the allocation process dynamic.

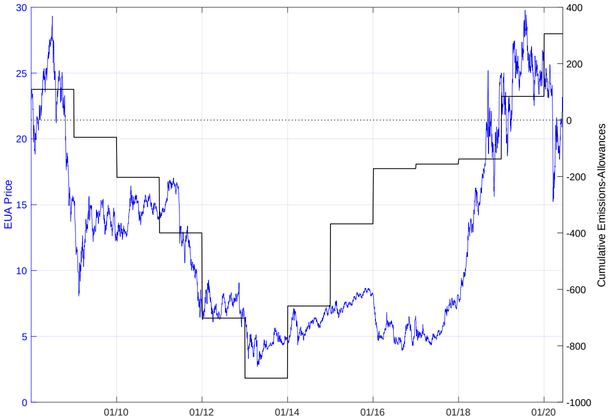


Figure: EUA price (€/tCO2) and difference between total verified emissions and total allocations (MtCO2).

Talk

- Investigation of optimal dynamic allocation schemes to reach a given expected emissions reduction over a finite time horizon.
- Our model is inspired by Kollenberg and Taschini (JEEM 2016, EER 2019).

Model

Emissions

- A regulator wishes to reduce the emissions of a set of N firms over a period of time $(0, T)$.
- Each firm i emissions follows the dynamics

$$E_t^i = \mu_i t + \sigma_i W_t^i, \quad \text{with} \quad W^i := \sqrt{1 - {k_i}^2}\,\tilde{W}^i + k_i \tilde{W}^0$$

  and $\tilde{W}_i$, $i = 0, \ldots, N$ independent, $\rho_{ij} := k_i k_j$.
- In the Business As Usual scenario (BAU), total expected emissions at time T are

$$\mathbb{E}\big[E_T\big] = N\bar{\mu}T, \quad \bar{\mu} := \frac{1}{N}\sum_{i=1}^{N} \mu_i.$$

- The regulator wishes to reduce the emissions to

$$L := \rho N \bar{\mu} T, \quad \rho \in (0, 1).$$

Dynamic allocations, bank accounts, abatement efforts and trading

- The regulator consider as possible instruments dynamic allocations of allowances.
- At time $t = 0$, the regulator opens a bank account for each i and credit (or debit) the account by the value x_0^i of allowances.
- The dynamics of the bank account of firm i is given by

$$dX_t^i = \tilde{a}_t^i dt + \beta_t^i dt - dE_t^{i,\alpha^i}, \quad dE_t^{i,\alpha^i} = -\alpha_t^i dt + \mu_i dt + \sigma_i dW_t^i.$$

- The process α^i is the abatement effort rate of firm i.
- The process β^i is the trading rate of firm i.
- The process $\tilde{a}_t^i$ is the allocation rate of allowances to firm i controled by the regulator.
- Dynamics of the bank accounts rewrites

$$dX_t^i = \left(\tilde{a}_t^i - \mu_i + \alpha_t^i + \beta_t^i\right) dt - \sigma_i dW_t^i.$$

Firms objective

- Abatement efforts of firm i comes at a cost

$$c_i(\alpha) := \underbrace{h_i\alpha}_{\text{prop. cost}} + \underbrace{\frac{1}{2}\frac{\alpha^2}{\eta_i}}_{\text{adjustment cost}}.$$

- For a given price process of allowances P and a given dynamic allocation scheme $(x_0, \tilde{a})$, each firm i wishes to solve

$$\inf_{\alpha^i,\beta^i} J^i(\alpha^i, \beta^i) := \mathbb{E}\Big[\int_0^T \Big(c_i(\alpha^i_t) + P_t\beta^i_t\Big)dt + \lambda\big(X^i_T\big)^2\Big],$$

and λ a parameter for the terminal bank account imbalances, reflects long-term social damages.

Remark

Possible to take into account market frictions on the trading of allowances $\frac{1}{2\nu}\beta^2$.

Market Equilibrium

- For a given allocation scheme $(x_0, \tilde{a})$, a market equilibrium is a vector of processes $(\hat{\alpha}, \hat{\beta})$ such that

$$J^i(\hat{\alpha}^i, \hat{\beta}^i) = \inf_{\alpha^i, \beta^i} J^i(\alpha^i, \beta^i), \quad \text{and} \quad \sum_{i=1}^{N} \hat{\beta}_t^i = 0.$$

Regulator's optimisation problem

Minimise total abatement costs and terminal penalty costs while ensuring a given emissions reduction.

$$\inf_{x_0, \tilde{a}} R(x_0, \tilde{a}) := \mathbb{E}\Big[\sum_{i=1}^{N} \int_0^T c_i(\hat{\alpha}_t^i) dt + \lambda(\hat{X}_T^i)^2\Big],$$

$$\mathbb{E}\Big[\sum_{i=1}^{N} E_T^{i,\hat{\alpha}^i}\Big] = L = \rho T N \bar{\mu}.$$

Remarks

- Full observability of abatement and trading rates and of economic shocks.
- It is a Stackelberg problem.

Some notations and useful variables

- Usefull to introduce and define the processes

$$a_t^i := \tilde{a}_t^i - \mu_i, \quad M_t^i := \mathbb{E}_t\Big[\int_0^T a_t^i dt\Big],$$

resp. the net allocation rate a^i and the total conditional expectation of the allocation M^i.

- And also the average quantities

$$\bar{M}_t := \frac{1}{N}\sum_{i=1}^N M_t^i, \quad \bar{H} := \frac{1}{N}\sum_{i=1}^N \eta_i h_i, \quad \bar{W}_t := \frac{1}{N}\sum_{i=1}^N \sigma_i W_t^i.$$

- Note that if a^i is a martingale,

$$M_t^i = \int_0^t a_s^i ds + a_t^i(T-t), \quad dM_t^i = (T-t)da_t^i.$$

Results

Equilibrium

For a given market net allocation scheme (x_0, a), the equilibrium price $\hat{P}$ is a martingale given by

$$d\hat{P}_t = -f(t)\Big(d\bar{M}_t - d\bar{W}_t\Big), \quad \hat{P}_0 = f(0)\Big(T\bar{H} - \bar{x}_0 - \bar{M}_0\Big),$$

$$\text{with} \quad f(t) := \frac{2\lambda}{1 + 2\lambda\bar{\eta}(T - t)}.$$

The abatement effort of firm i is unique and given by:

$$\hat{\alpha}^i_t = \eta_i\big(\hat{P}_t - h_i\big).$$

Comments

- If firms expect that more allowances are going to be injected ($d\bar{M}_t > 0$), the price $\hat{P}$ decreases.
- If firms experience a positive economic shock ($d\bar{W}_t > 0$), the price increases.

Why?

- Take firm i criteria

$$J^i(\alpha^i,\beta^i) := \mathbb{E}\Big[\int_0^T \Big(h_i\alpha_t^i + \frac{1}{2}\frac{(\alpha_t^i)^2}{\eta_i} + P_t\beta_t^i\Big)dt + \lambda\big(X_T^i\big)^2\Big].$$

- First-order conditions w.r.t. α^i and β^i are

$$\underbrace{h_i + \frac{1}{\eta_i}\alpha_t^i}_{\text{marginal cost}} + \underbrace{2\lambda\mathbb{E}_t\big[X_T^i\big]}_{\text{marginal penalty}} = 0, \quad P_t + 2\lambda\mathbb{E}_t[X_T^i] = 0.$$

- Thus, the price satisfies

$$P_t = -\frac{2\lambda}{N}\sum_{i=1}^N \mathbb{E}_t\big[X_T^i\big] =: -2\lambda\mathbb{E}_t\big[\bar{X}_T\big].$$

- And the α^i are martingales satisfying

$$\alpha_t^i = \eta_i(P_t - h_i), \quad \bar{\alpha}_t = \bar{\eta}P_t - \bar{H}, \quad d\bar{\alpha}_t = \bar{\eta}dP_t.$$

Why? (cont.)

- Since $\bar{\alpha}$ is a martingale,

$$\mathbb{E}_t\left[\bar{X}_T\right] = \mathbb{E}_t\left[\bar{x}_0 + \int_0^T \left(\bar{a}_t + \bar{\alpha}_t\right) dt - \bar{W}_T\right],$$
$$= \bar{x}_0 + \bar{M}_t + \int_0^t \bar{\alpha}_s ds + (T-t)\bar{\alpha}_t - \bar{W}_t.$$

- Thus,

$$dP_t = -2\lambda d\mathbb{E}_t\left[\bar{X}_T\right] = -2\lambda\Big[d\bar{M}_t + (T-t)d\bar{\alpha}_t - d\bar{W}_t\Big].$$

- Subtitution of $d\bar{\alpha}_t = \bar{\eta} dP_t$ provides

$$dP_t = -\frac{2\lambda}{1 + 2\lambda\bar{\eta}(T-t)}\Big[d\bar{M}_t - d\bar{W}_t\Big].$$

Consequences for optimal regulation

- Total expected emissions only depend on average effort rate $\bar{\alpha}$ and since it is a martingale, we have

$$\mathbb{E}\big[N\bar{E}_T\big] = NT\big(\bar{\mu} - \bar{\alpha}_0\big) = NT\big(\bar{\mu} - \bar{\eta}\hat{P}_0 + \bar{H}\big), \quad \bar{H} = \frac{1}{N}\sum_{i=1}^{N} \eta_i h_i.$$

- To achieve a reduction by a factor ρ it should hold that

$$\hat{P}_0 = \frac{\bar{H}}{\bar{\eta}} + (1-\rho)\frac{\bar{\mu}}{\bar{\eta}}.$$

Comment

The average price is made of two components.

- The average of the linear part of the marginal abatement cost
- The term taking into account the adjustment cost, the growth rate of emissions and the ambition of the regulation.
- Recall Gollier (2020) carbon price puzzle talk at FiME seminar.

Consequences for optimal regulation

- The expression of $\hat{P}_0$ says that it is fully detemined by $\bar{x}_0 + \bar{M}_0$.
- Thus, to achieve a reduction by a factor ρ, one should peak $\bar{x}_0 + \bar{M}_0$ such that:

$$\bar{x}_0 + \bar{M}_0 = -\frac{1}{2\lambda\bar{\eta}}\left[\bar{H} + \big(1 + 2\lambda\bar{\eta}T\big)(1-\rho)\bar{\mu}\right] =: \ell(\rho) < 0.$$

Comment

- Suppose that the regulator does not want to add or withdraw on average allowances.
- Thus, $\bar{M}_0 = 0$, and $\bar{x}_0 < 0$.
- On average, the bank accounts should be endowed with a debt.

Regulation optimisation problem rephrased

Using the fact that

$$\hat{P}_T = -2\lambda \mathbb{E}_T\left[\bar{X}_T\right] = -2\lambda \bar{X}_T$$

The regulator problems is now

$$\inf_{a,x_0} \mathbb{E}\Big[\sum_{i=1}^{N} \int_0^T \Big(h_i \eta_i (\hat{P}_t - h_i) + \frac{1}{2}\eta_i (\hat{P}_t - h_i)^2\Big) dt + \frac{(\hat{P}_T)^2}{4\lambda}\Big]$$

$$d\hat{P}_t = -f(t)(d\bar{M}_t - d\bar{W}_t), \quad \hat{P}_0 = \frac{\bar{H}}{\bar{\eta}} + (1-\rho)\frac{\bar{\mu}}{\bar{\eta}}, \quad \bar{x}_0 + \bar{M}_0 = \ell(\rho).$$

Remarks

- The M^i are martingales. They can be written

$$M_t^i = M_0^i + \int_0^T \gamma_t^i \cdot dW_t, \quad \gamma^i := (\gamma^{i,k}).$$

- Thus, the regulator's problem is a stochastic LQ problem where the controls γ^k only appears in the volatility of the state variable which is the price $\hat{P}$.

$$\inf_{a,x_0} \mathbb{E}\Big[\sum_{i=1}^N \int_0^T \Big(h_i \eta_i (\hat{P}_t - h_i) + \frac{1}{2} \eta_i (\hat{P}_t - h_i)^2 \Big) dt + \frac{(\hat{P}_T)^2}{4\lambda} \Big]$$

$$d\hat{P}_t = -\frac{1}{N} f(t) \Big[\sum_{i=1}^N \underbrace{\Big\{ \sum_{k=1}^N \gamma_t^{k,i} - \sigma^i \Big\}}_{g_i(\gamma^i)} dW_t^i \Big].$$

- Thus, the optimal controls γ^i are to be choosen to minimise the volatility of the price

Remarks (cont.)

- Volatility minimisation is achieved for instance by taking
$$\gamma^{i,i} = \sigma_i, \quad \gamma^{i,k} = 0, \quad i \neq k.$$
- And, knowing M^i, one can find a net allocation rate a^i in the class of martingales using
$$dM^i = \sigma_i dW_t^i = (T - t) da_t^i.$$
- Initial condition of the total net expected allocations M_0^i is only constrained to satisfy
$$\bar{x}_0 + \bar{M}_0 = \ell(\rho).$$

Optimal regulations

(i) The solutions to the regulator optimisation problem are non-unique and characterised by the minimisation of the volatility of the price and the condition that $\bar{x}_0 + \bar{M}_0 = \ell(\rho)$.

(ii) The net allocations given by

$$a_t^{i,*} = \int_0^t \frac{\sigma_i}{T-t} dW_s^i, \quad x_0^i = \ell(\rho), \quad \forall i = 1, \ldots, N,$$

form a solution.

(iii) The equilibrium price and abatement efforts are constant given by

$$\hat{P}_0 = \frac{\bar{H}}{\bar{\eta}} + (1-\rho)\frac{\bar{\mu}}{\bar{\eta}}, \quad \hat{\alpha}_0^i = \eta_i(\hat{P}_0 - h_i).$$

A specific allocation scheme

Consider the proposed allocation scheme

$$\tilde{a}_t^{i,*} = \mu_i + a_t^{i,*} = \mu_i + \int_0^t \frac{\sigma_i dW_t^i}{T-t}, \quad x_0^i = \ell(\rho).$$

Comments and remark

The regulator

- provides an equal debt on all firms,
- compensate each firm from the emission trend of the BAU and of the economic shocks that affect it.

There are no reasons in this regulation framework to prefer this allocation scheme to any other optimal allocation scheme.

Where is the benefit
of a dynamic reallocation scheme
compared to a simple static initial allocation?

Static allocation scheme (EU TS Phase I to III)

- The static allocation scheme corresponds to

$$0 = \tilde{a}^i = \mu_i + a^i, \quad \bar{M}_0 = -\bar{\mu} T, \quad x_0^i = \ell(\rho) + \bar{\mu} T.$$

- For sake of computation, suppose all firms endure the same adjustement cost parameter $\eta_i = \eta$.
- Denote Δ the difference between the social cost with a static allocation and the social cost under an optimal dynamic allocation.

Static allocation scheme (EU TS Phase I to III)

- We have

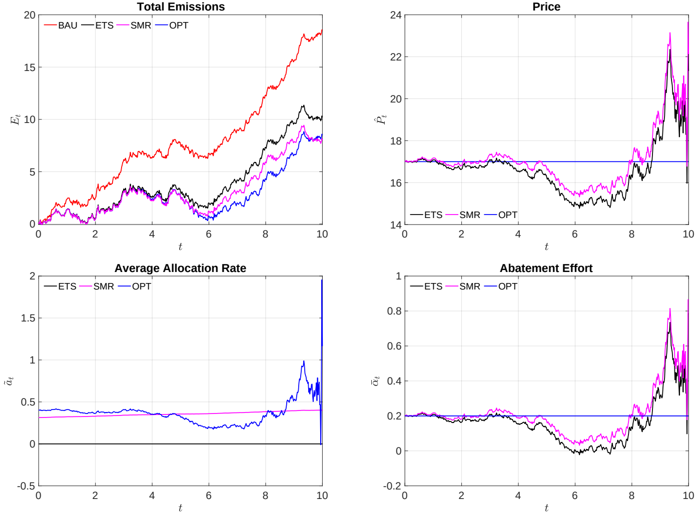
$$\Delta := \frac{\phi^2}{N}\frac{1}{2\eta}\ln\left[1 + 2\lambda\eta T\right], \qquad \phi^2 := \sum_{i=1}^{N}\sigma_i^2 + 2\sum_{i<j}\rho_{ij}\sigma_i\sigma_j.$$

- If the world is deteministic ($\phi = 0$) or perfectly flexible ($\eta \to \infty$), there is no interest in dynamic allocation.
- If there are no common shocks, when $N \to \infty$, the per unit difference cost Δ/N goes to zero, making also dynamic schemes useless.

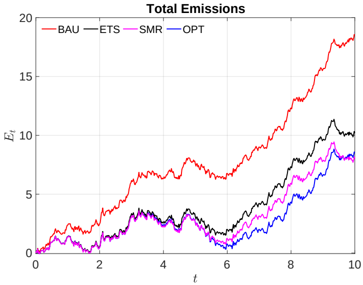
- Dynamic allocation provides insurance to firms from common economic uncertainty inducing costly adjustment.

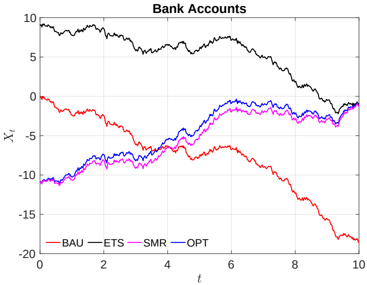
Illustration of the dynamics

- In $T = 10$ years,
- the regulator wants to reduce the total expected emissions by $\rho = 0.5$,
- in a market where the average growth rate of emissions is $N\bar{\mu} = 2$ Gt/year,
- with a volatility of $\sigma_i = 0.6/\sqrt{N}$ Gt/year and per firm,
- and average abatement cost $\bar{h} = 15$ €/t,
- and equal adjustment cost $\eta = 10^8$ t^2/€,
- and a equal dependence on the common shocks of $k_i = 0.8$
- and a terminal penalty parameter of $\lambda = 5\,10^{-8}$ €/t^2.



Passive regulator → frenzy firms   Active regulator → serene firms





Same reduction target reached with quiet similar trajectories but with different bank accounts trajectories and total expected costs (117 billion € for optimal allocation and 261 for static allocation).

Conclusions

- Optimal dynamic allocations of allowances lead to price volatility minimisation.
- Active regulation provides insurance to firms from costly adjustement to economic shocks.
- Non-uniqueness of dynamic allocation schemes allow for dealing with multiple objectives.

Perspectives

- Stationary case.
- Non-observability of reduction efforts and moral hazard.
- Innovation financing.
- Producers/consumers.